Refresh on earthquake location: a quick tour behind the scenes

Dino Bindi

GFZ-Potsdam
bindi@gfz-potsdam.de

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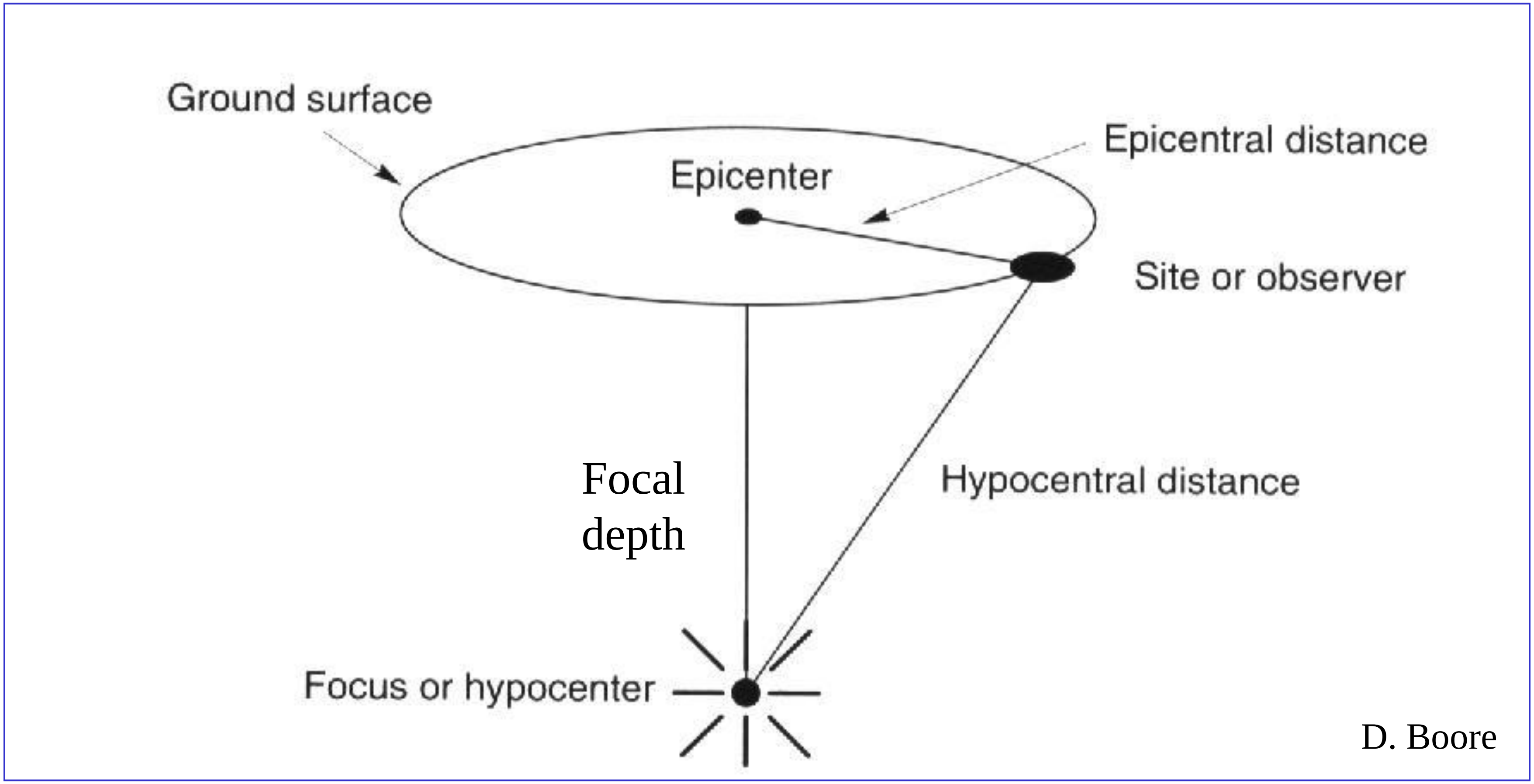
EARTHQUAKE LOCATION

aka The fantastic 4
Thing: Depth
Invisible women: Origin Time
Mr. Fantastic & Human Torch: lat & lon

To locate an earthquake we need to determine the spatial coordinates of the hypocentre and the origin time (**four unknowns**)

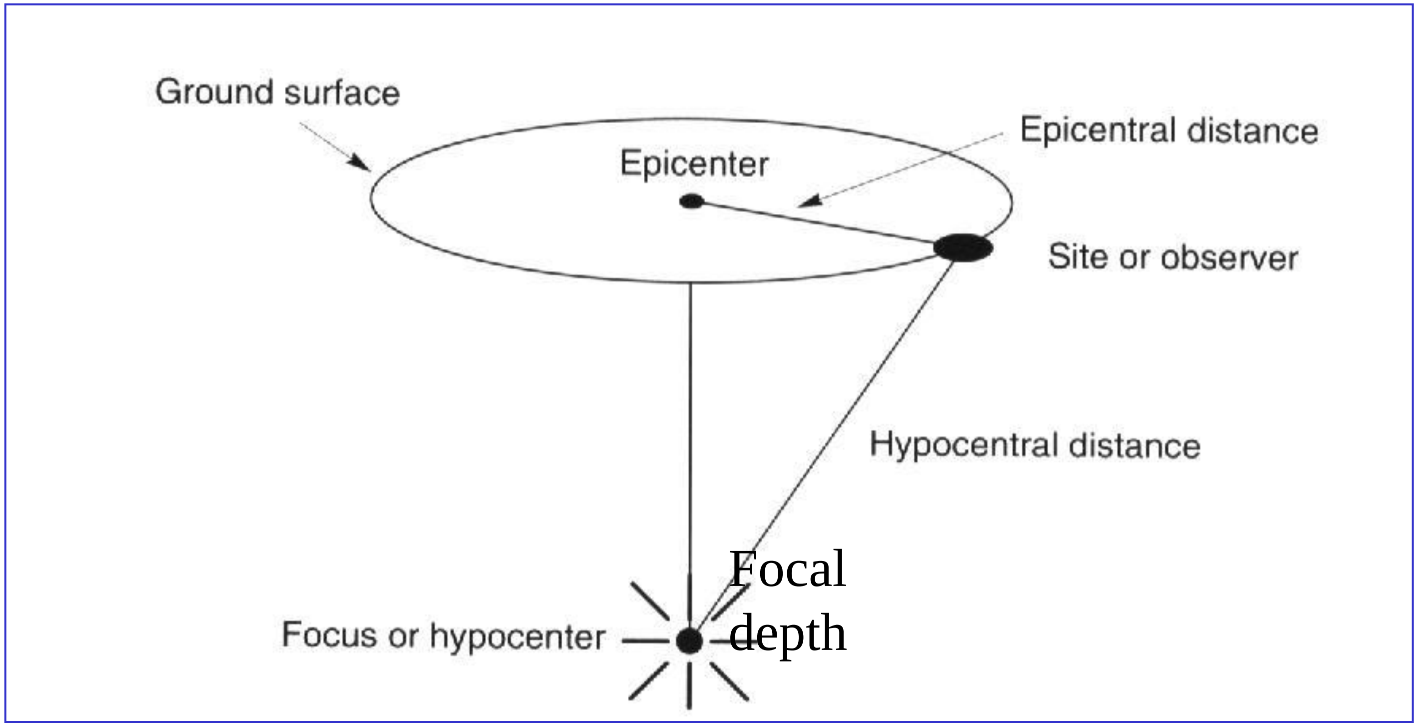
$$\left.\begin{array}{l}\textbf{Epicentral Coordinates} \\ \textbf{(latitude,longitude)} \\ \\ \textbf{Focal depth, } h \textbf{ (km)}\end{array}\right\} \textbf{SPACE}$$

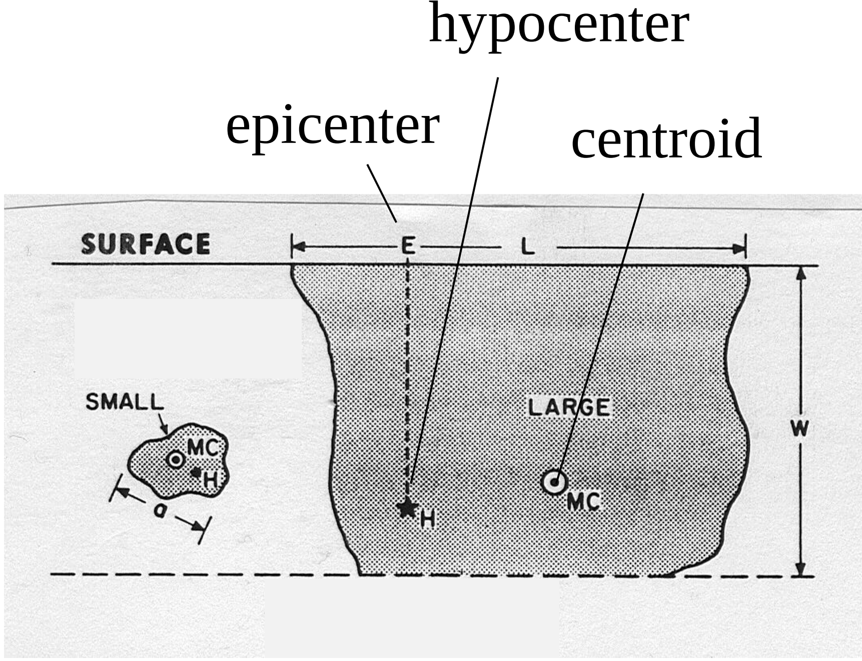
Origin time, t_o **TIME**



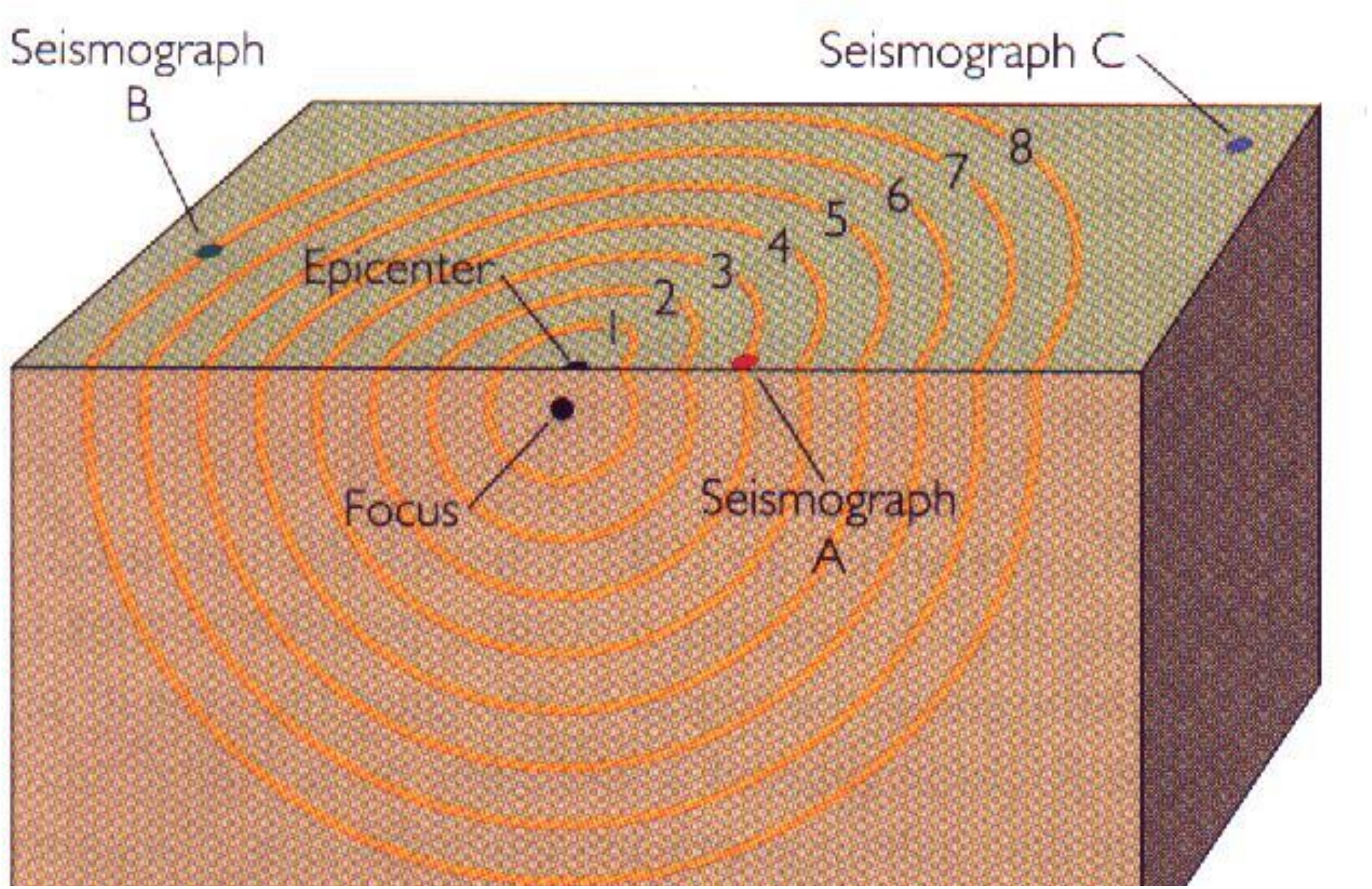
D. Boore

Note: for large earthquake, the fault dimension could be several hundred of km. In that case, what does hypocenter mean? Since the hypocenter is generally determined considering the arrival times of phases generated by the initial rupture process, the hypocenter will be located close to the region where the rupture initiated. This is true when P and S arrival times are considered since their velocity of propagation is generally higher than the velocity of propagation of the rupture over the fault.


Ground surface
Epicentral distance
Epicenter
Site or observer
Hypocentral distance
Focal depth
Focus or hypocenter


hypocenter
epicenter
centroid
SURFACE
E
L
W
SMALL
MC
H
a
LARGE
H
MC

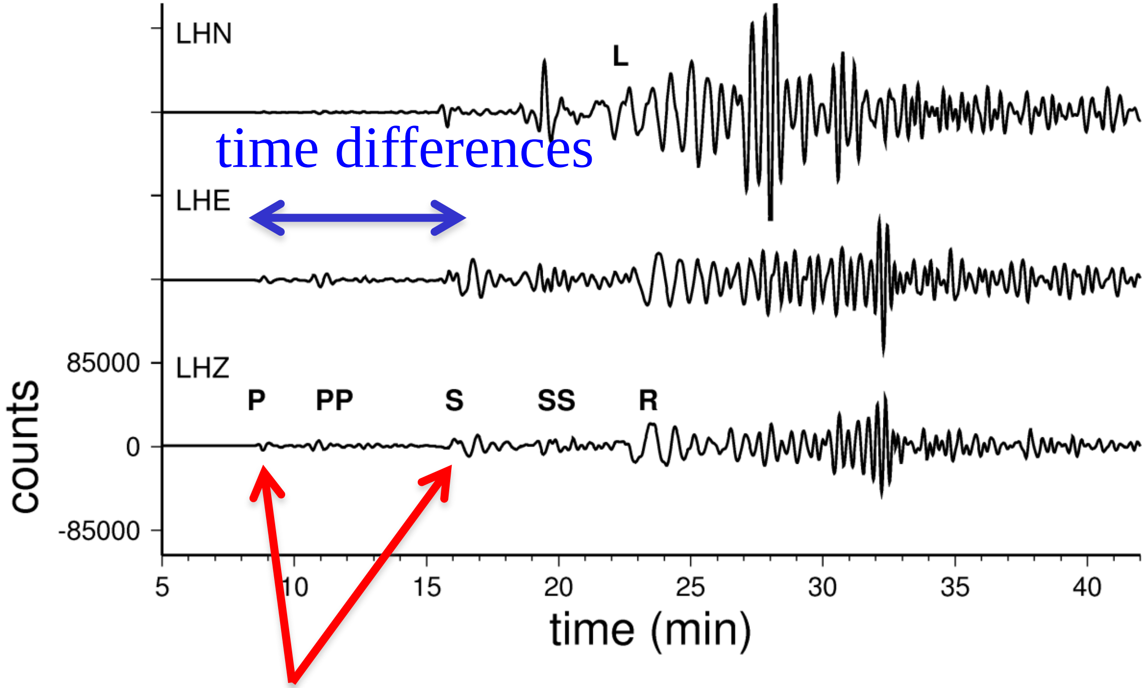
Basic idea: look at relative arrival times of phases at different stations (waves will arrive at A, then B, and then C)



When locating an earthquake, the velocity model is assumed to be known (in the observatory practice)

1. Earthquake location (point source, 4 parameter)

Pakistan, 8 Okt. 2005, MS 7.8, LP 30 S



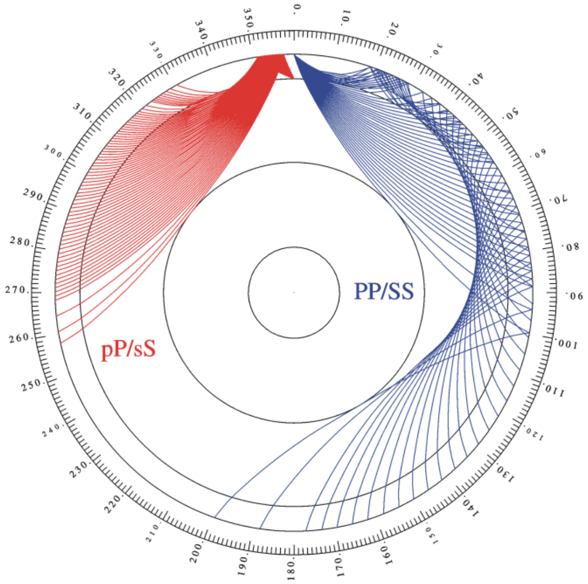
time differences

arrival times

Earthquake location: phase picking and association

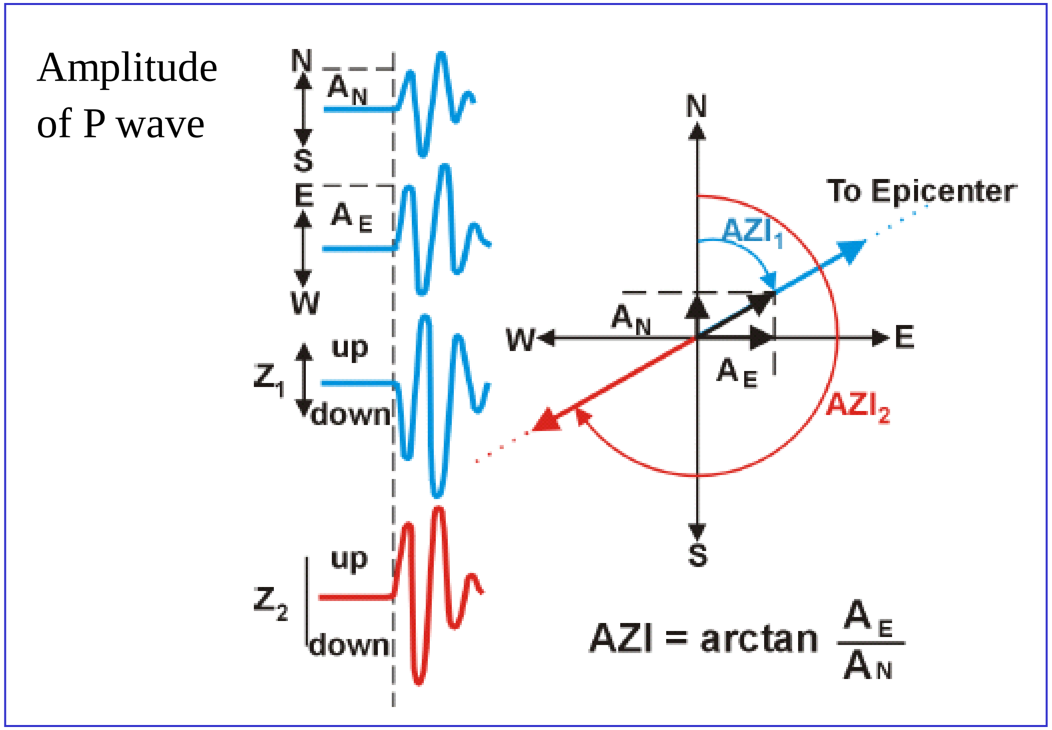
P: direct P wave
S: direct S wave
pP: near source, surface reflected P wave (depth phase)
PP: surface reflected P wave
PmP: Moho reflected P wave
Pn: upper mantle refracted P wave
.....

PP/SS and pP/sS



Before facing the problem of determining the earthquake locations from arrival times at different stations, what we can do when only one three component station is available?

Determination of the azimuth

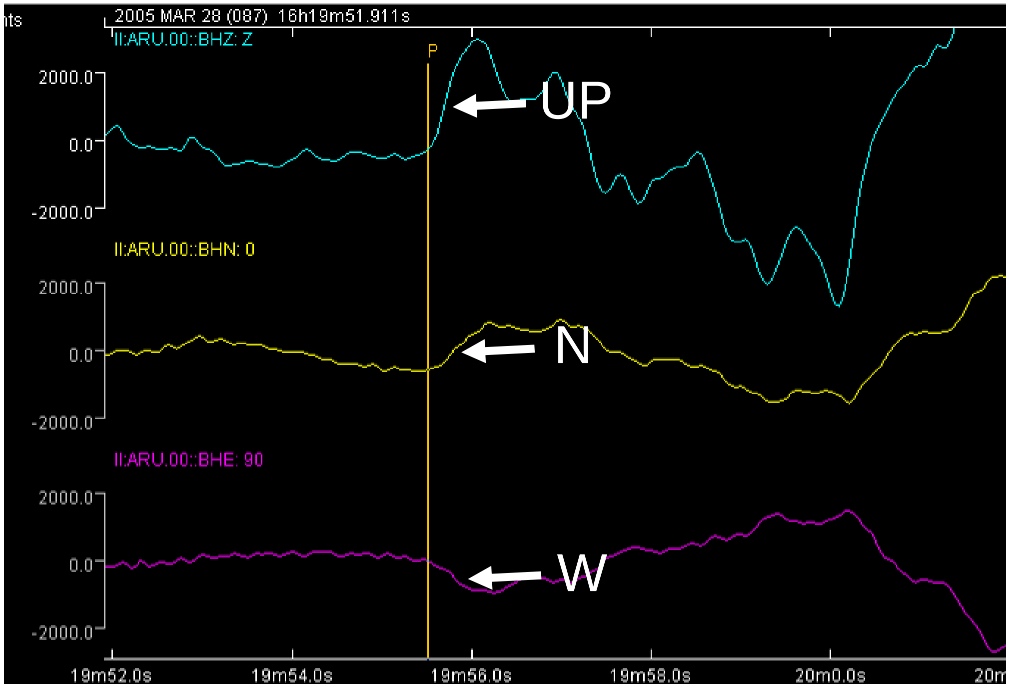


$$AZI = \arctan \frac{A_E}{A_N}$$

With AN and AE we can determine the radial directions. Using the polarity of the vertical component we can fix the ambiguity of π.

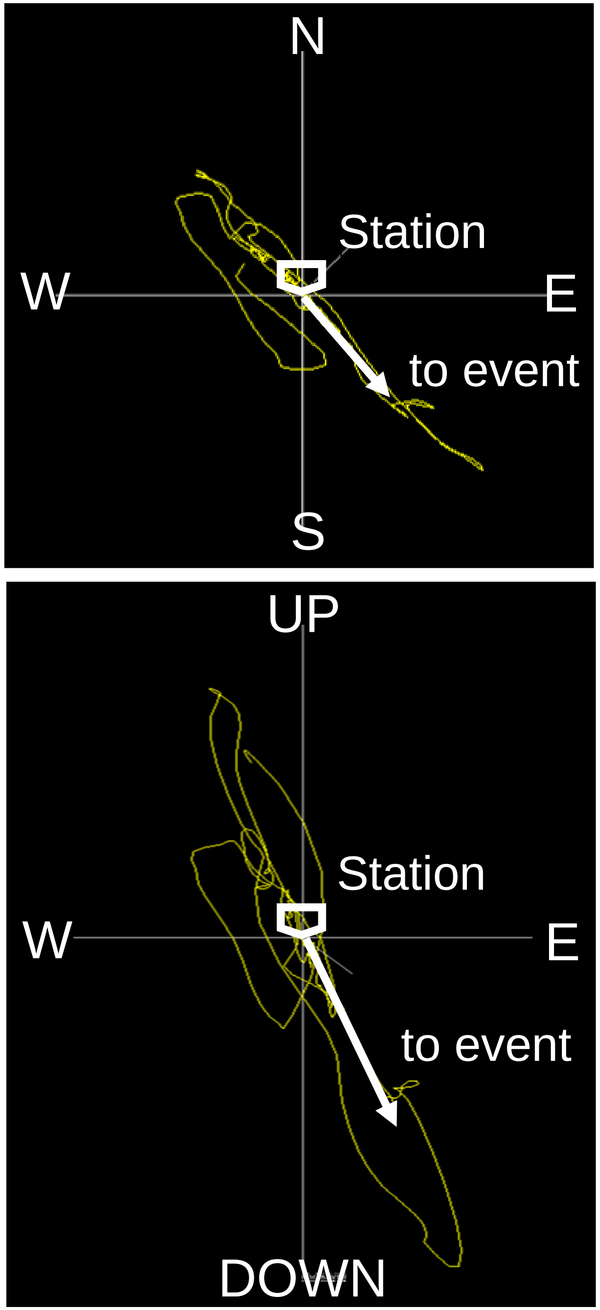
Single station method

The ratio of movement on the horizontal components gives the azimuth



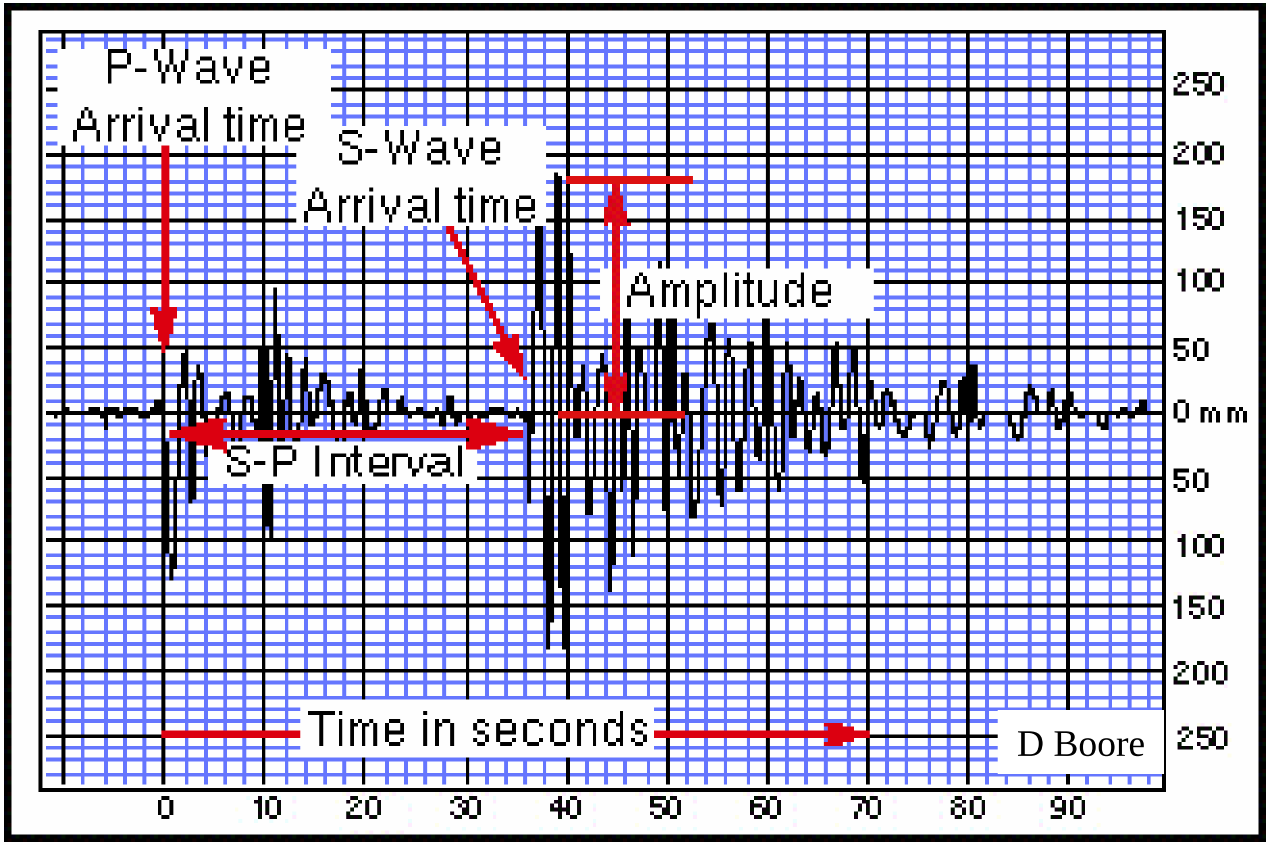
And the distance? →

Particle motion – P wave

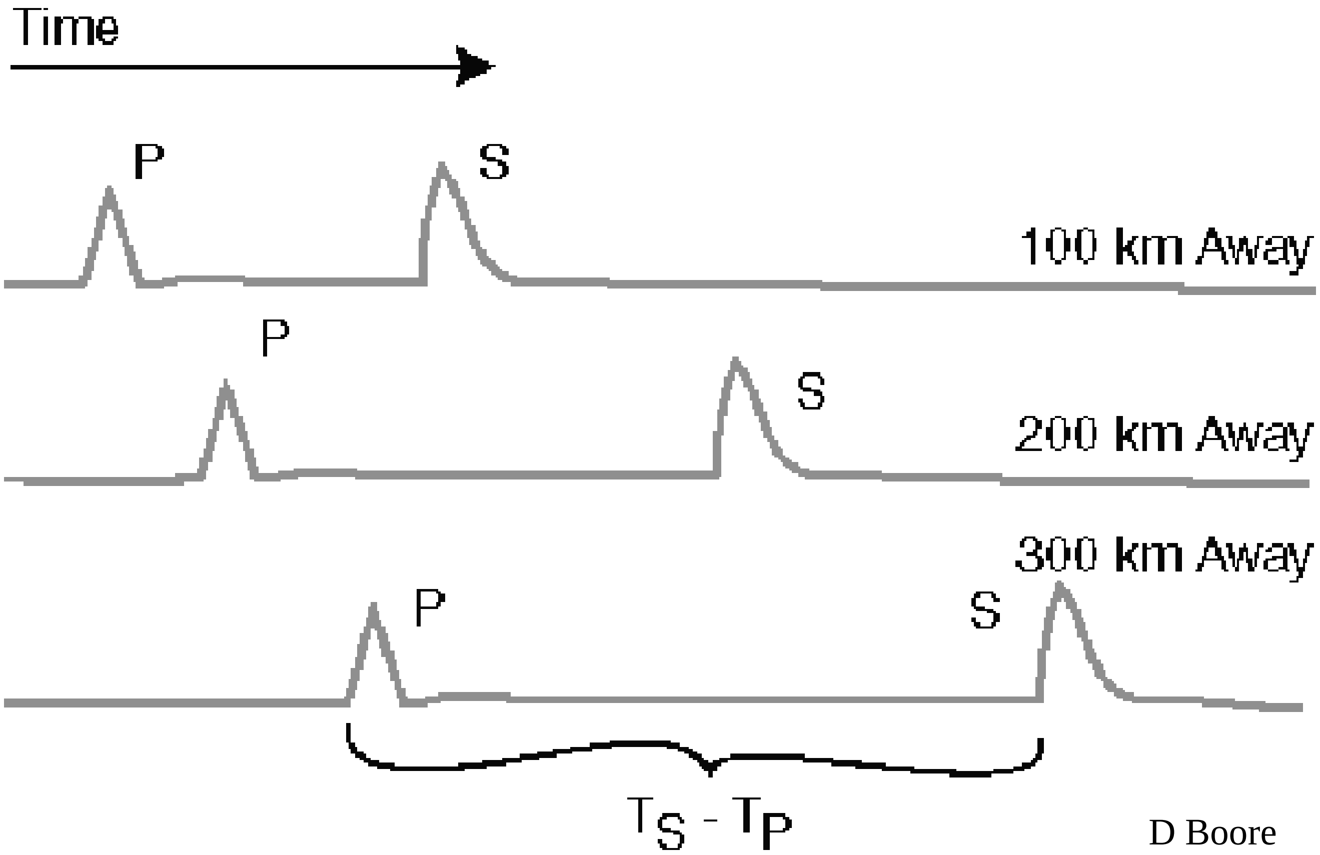


Courtesy of W. Mooney

The distance can be estimated from the difference of the arrival times of P and S waves.



S-P Time Example



D Boore

$$t_p = t_0 + D / v_p \qquad t_s = t_0 + D / v_s \qquad (1)$$

where t_p and t_s are the P and S waves arrival times, respectively, and D the distance

$$D = (t_s - t_p)\frac{v_p v_s}{v_p - v_s} \qquad (2)$$

This equation can be applied only to direct crustal phases, as Pg and Sg, i.e for distances not exceeding 150-250 km (depending on the crust thickness and the focal depth). The cross-over distance (for larger distances the first arrivals are Pn and Sn) can be approximated as

$$x_{co} = 2z_m \left\{ \left(v_m - \bar{v}_p \right) \left(v_m + \bar{v}_p \right) \right\}^{-1/2}$$

where $\bar{v}_p$ is the average crustal P wave velocity and v_m is the P velocity at the Moho depth, and z_m is the crustal thickness. For velocity equal to 6 km/s and 8 km/s, respectively, we obtain x_{co}~5 zm (as "**rule of thumb**")

“rule of thumbs”for local earthquakes

Assuming Vp/Vs=1.73 (Poisson condition):

$$D=(t_S-t_P)x8.0 \quad \text{for } V_p=5.9 \text{ km/s (for average crustal condition)}$$

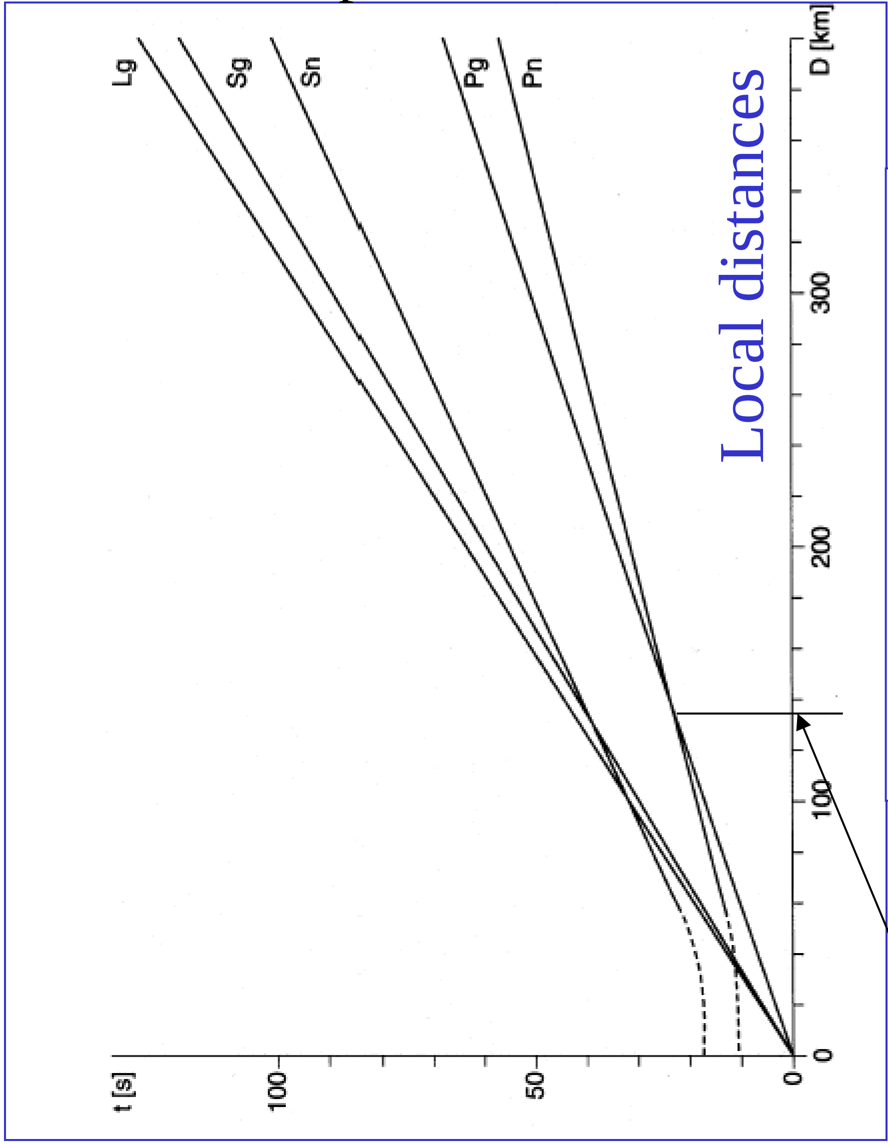
$$D=(t_S-t_P)x9.0 \quad \text{for } V_p=6.6 \text{ km/s (e.g. continental shields)}$$

If an estimate of the Poisson ratio is known, then the previous rules can be improved.

In the case of Pn and Sn, the “rule of thumb” is:

$$D=(t_{Sn}-t_{Pn})x10.0 \quad \text{(D in km)}$$

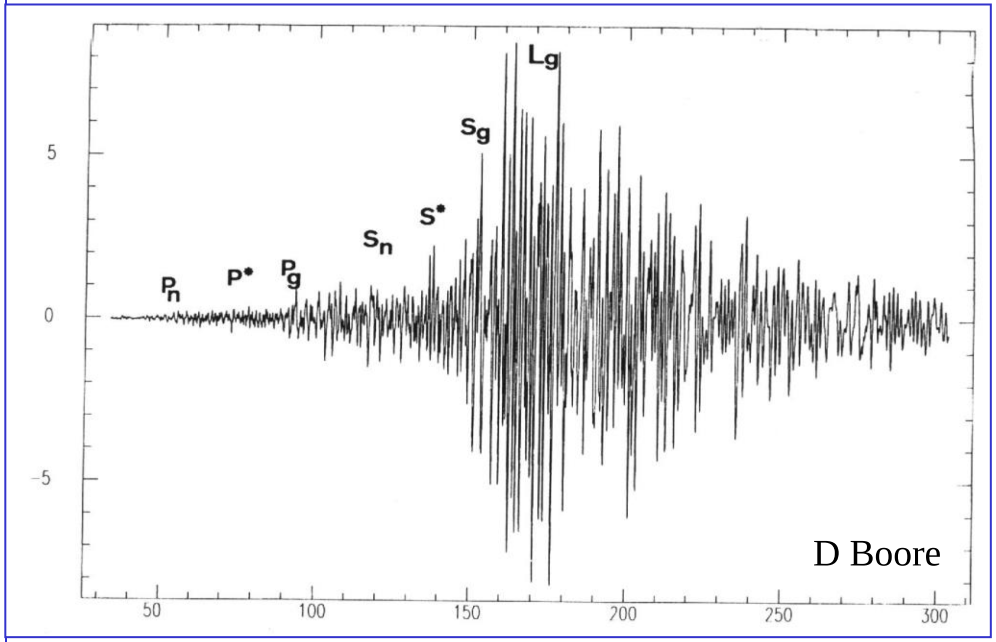
Travel time curves for the area can be used to exploit the arrival times of several phases to determine D



Local distances

[Km]

seconds



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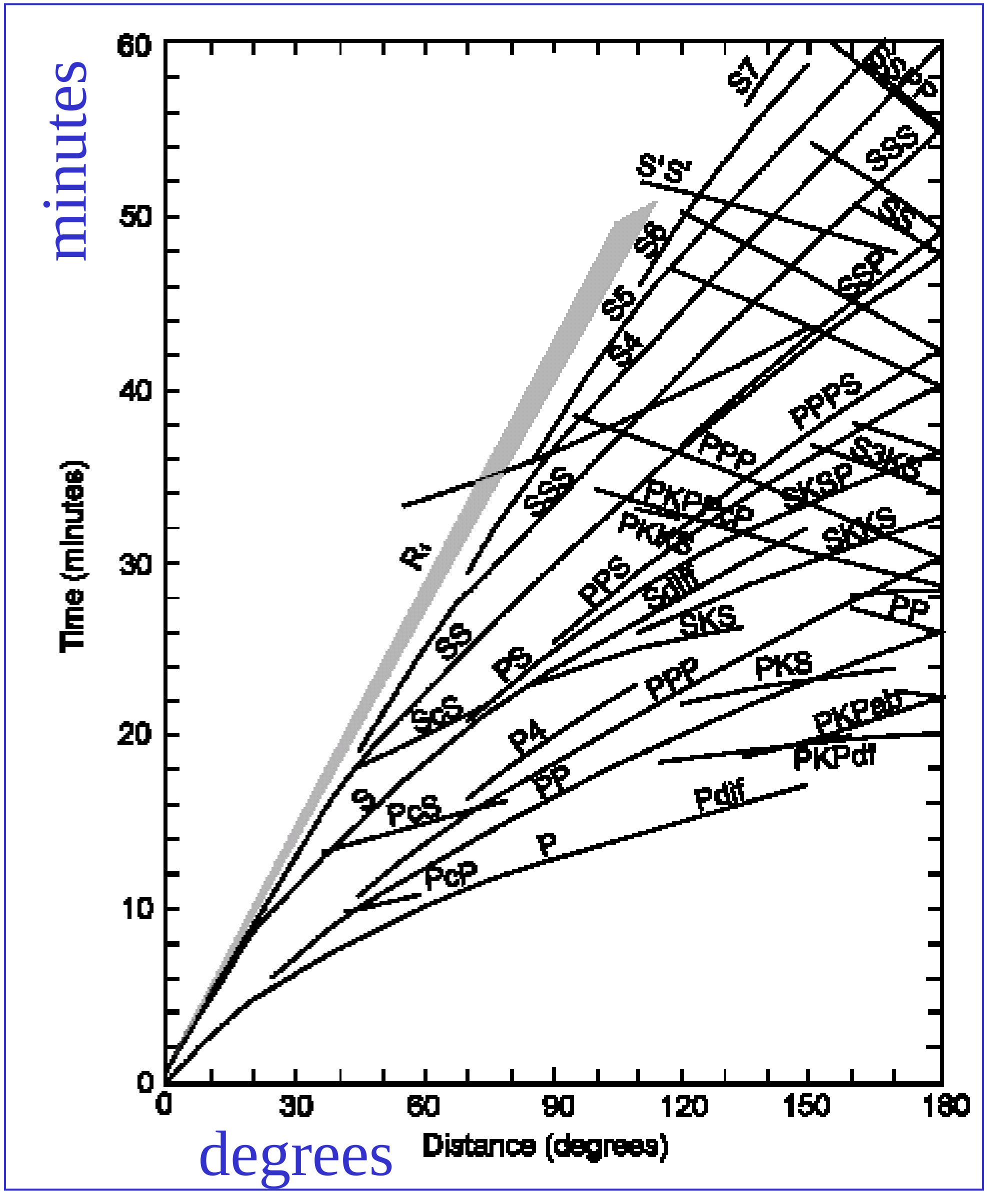
Cross-over distance

Teleseismic distances



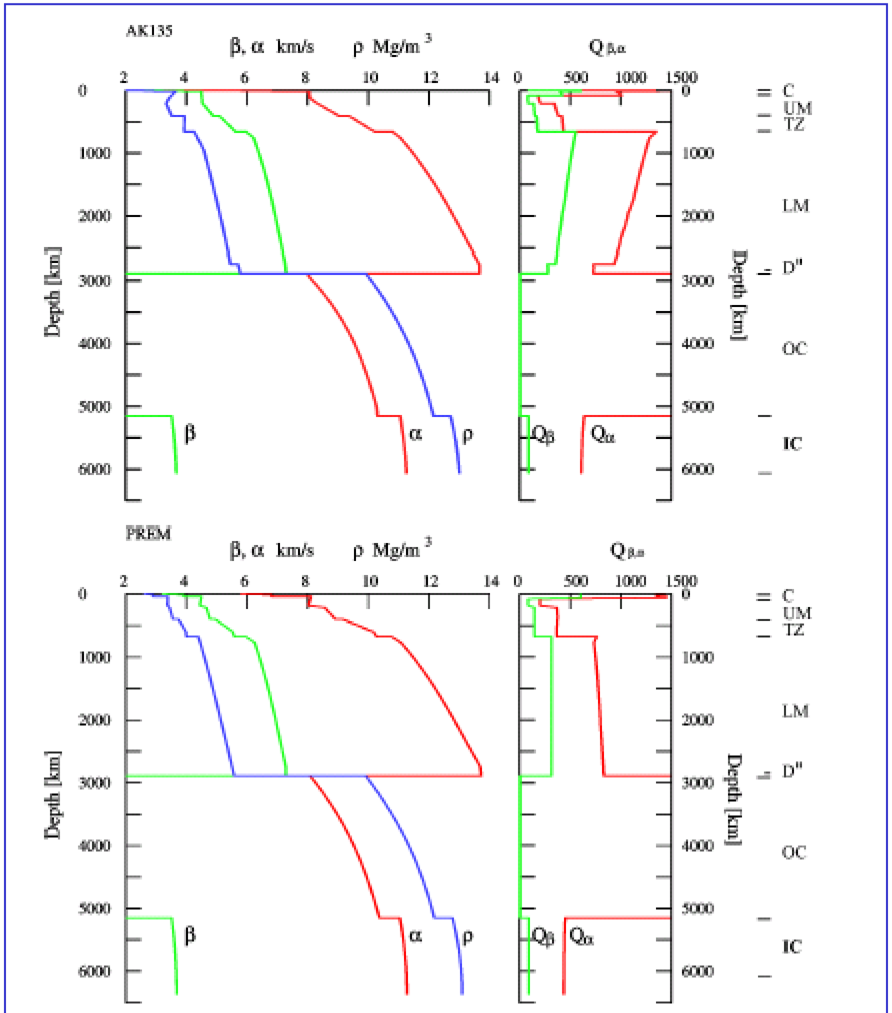
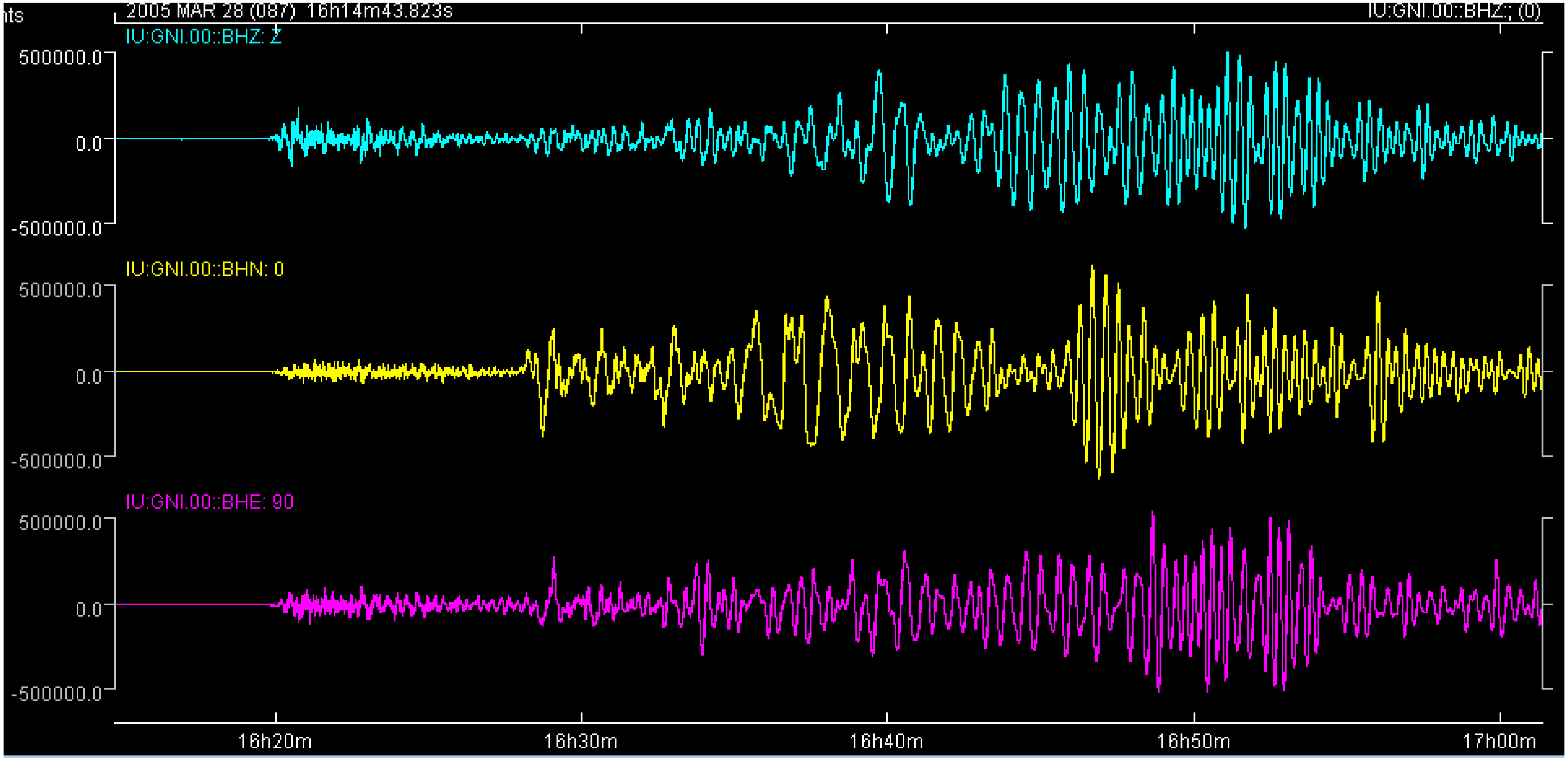


Fig. 2.53 Radial symmetric reference models of the Earth. Top: AK135 (seismic wave speeds according to Kennett et al. (1995), attenuation parameters and density according to Montagner and Kennett (1996); Bottom: PREM (Dziewonski and Anderson, 1981). α - and β: P- and S-wave velocity, respectively; ρ - density, Q_α and $Q_\beta = Q_\mu$ - "quality factor" Q for P and S waves. Note that wave attenuation is proportional to 1/Q. The abbreviation on the outermost right stand, within the marked depth ranges, for: C – crust, UM – upper mantle, TZ – transition zone, LM – lower mantle, D"-layer, OC – outer core, IC – inner core.

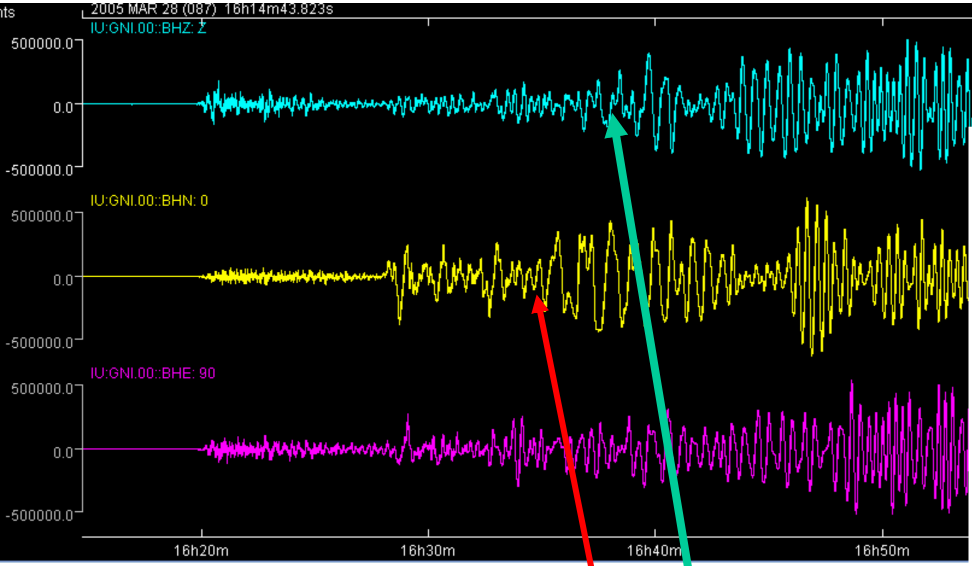
Example for a teleseism



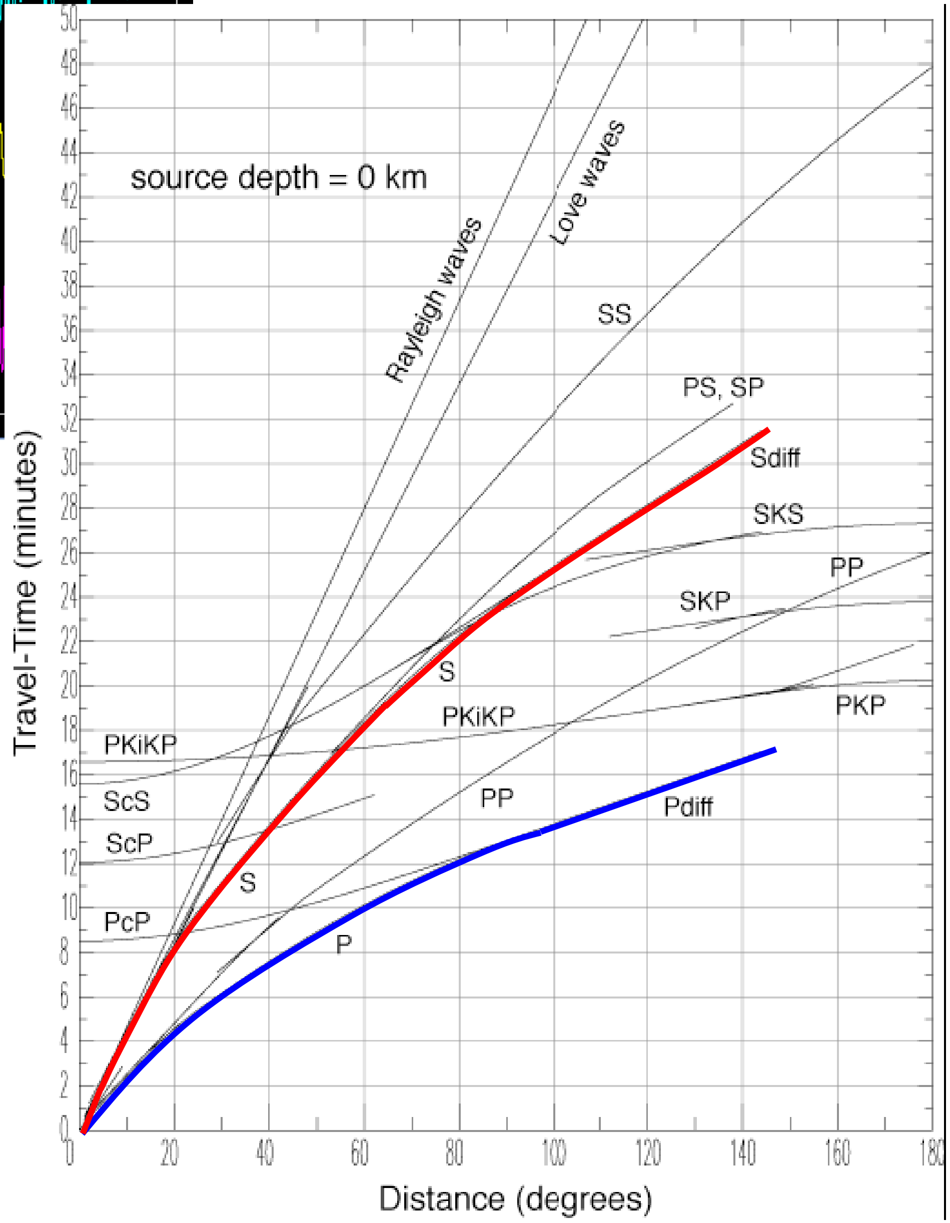
March 28, 2005 M8.7 Sumatra earthquake, as recorded at GNI station in Armenia (60 Degrees from the epicenter)

ts-tp is about 8 minutes

Courtesy of A. Kelly

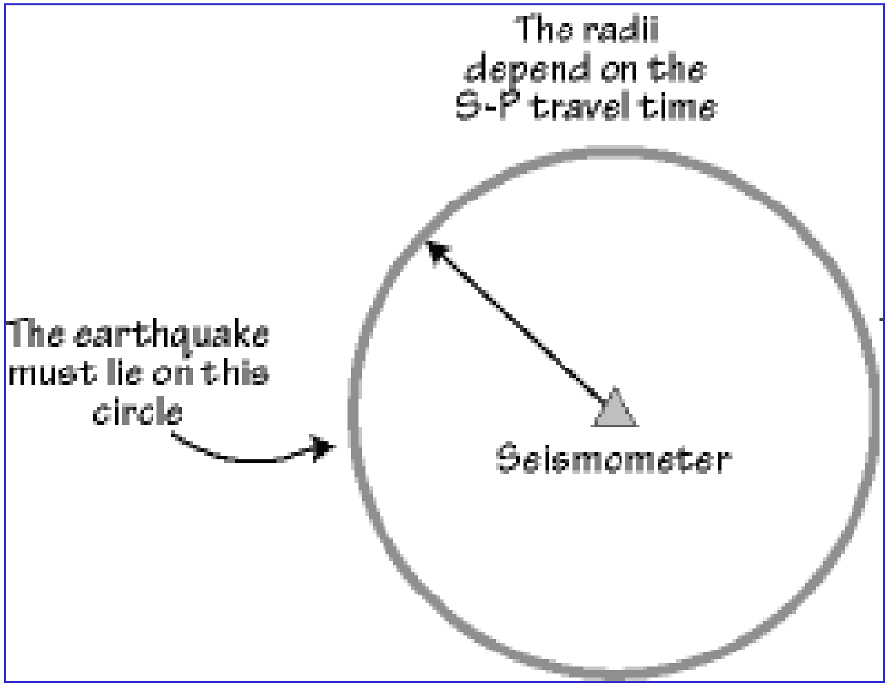


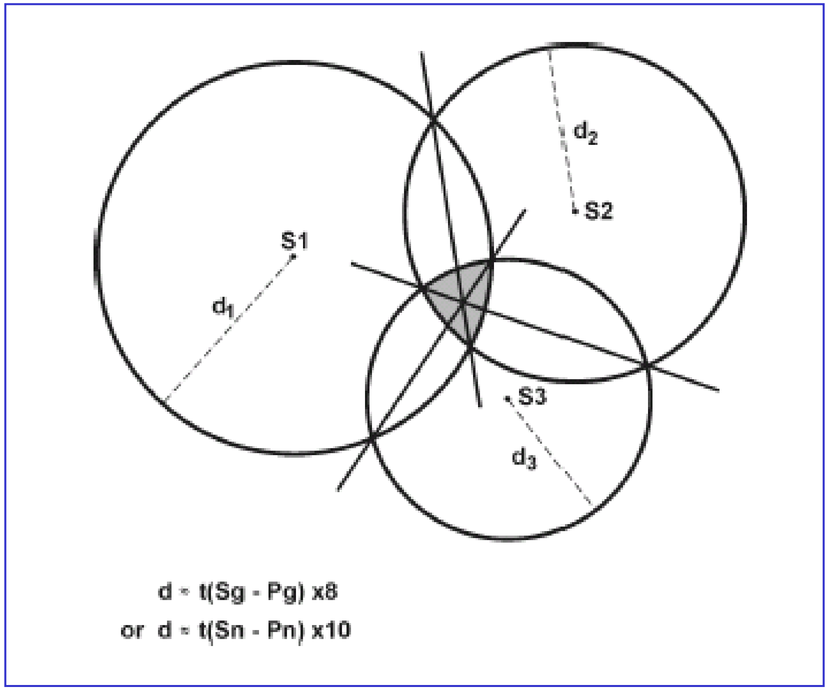
ts-tp is about 8 minutes
→ the travel time curves provide a distance of 60° (ok!)
at 60° the Love arrives approximately here
and the Rayleigh here



PREM model, Dziewonski & Anderson, 1981

If more than 1 station is available (at least 3), then the epicenter can be estimated using a "triangulation" procedure:



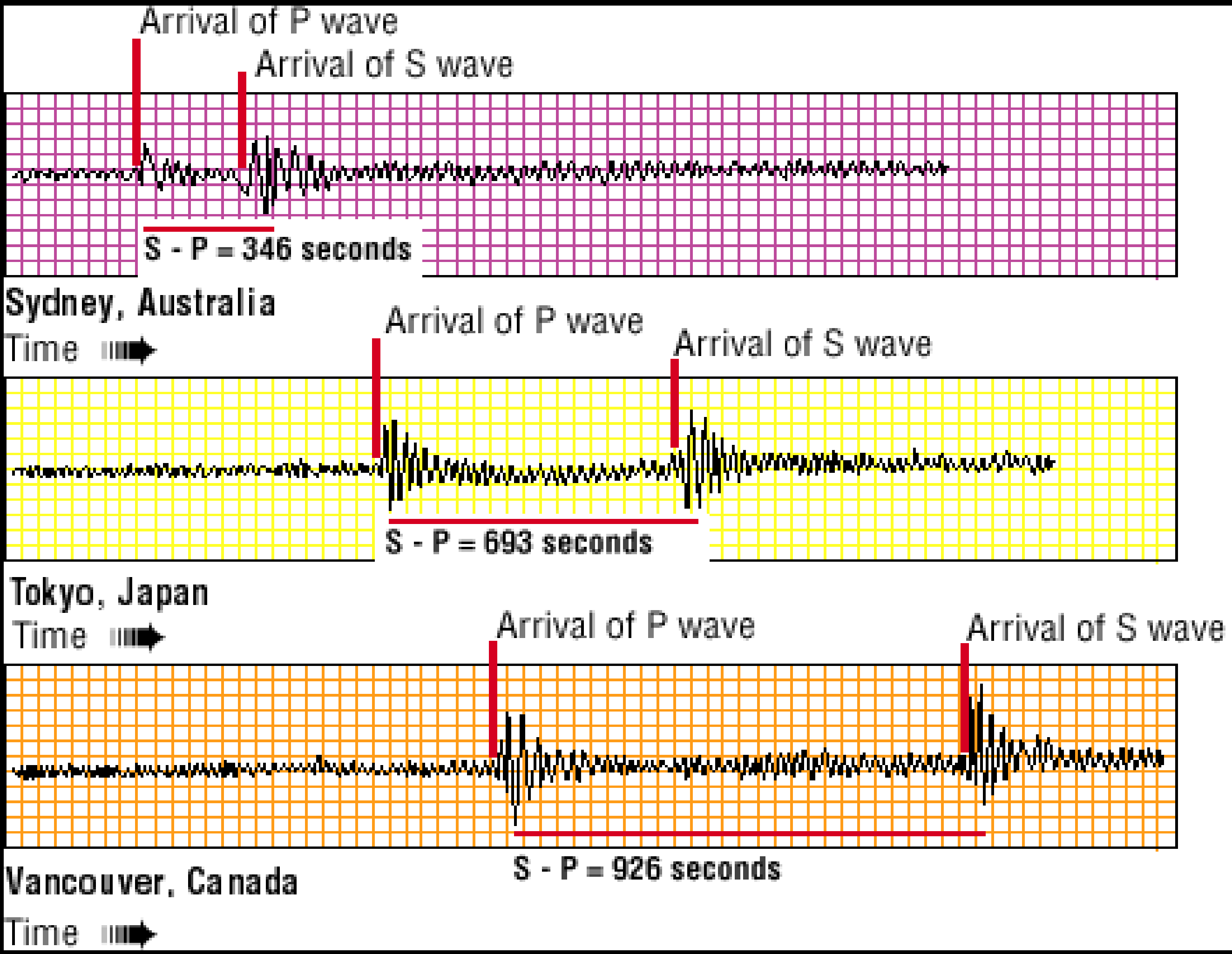


Earthquakes

Here we have measured the time intervals between the arrival of the P and the S waves at each of our three seismic locations:

	S-P interval
Sydney	346 s
Tokyo	693 s
Vancouver	926 s

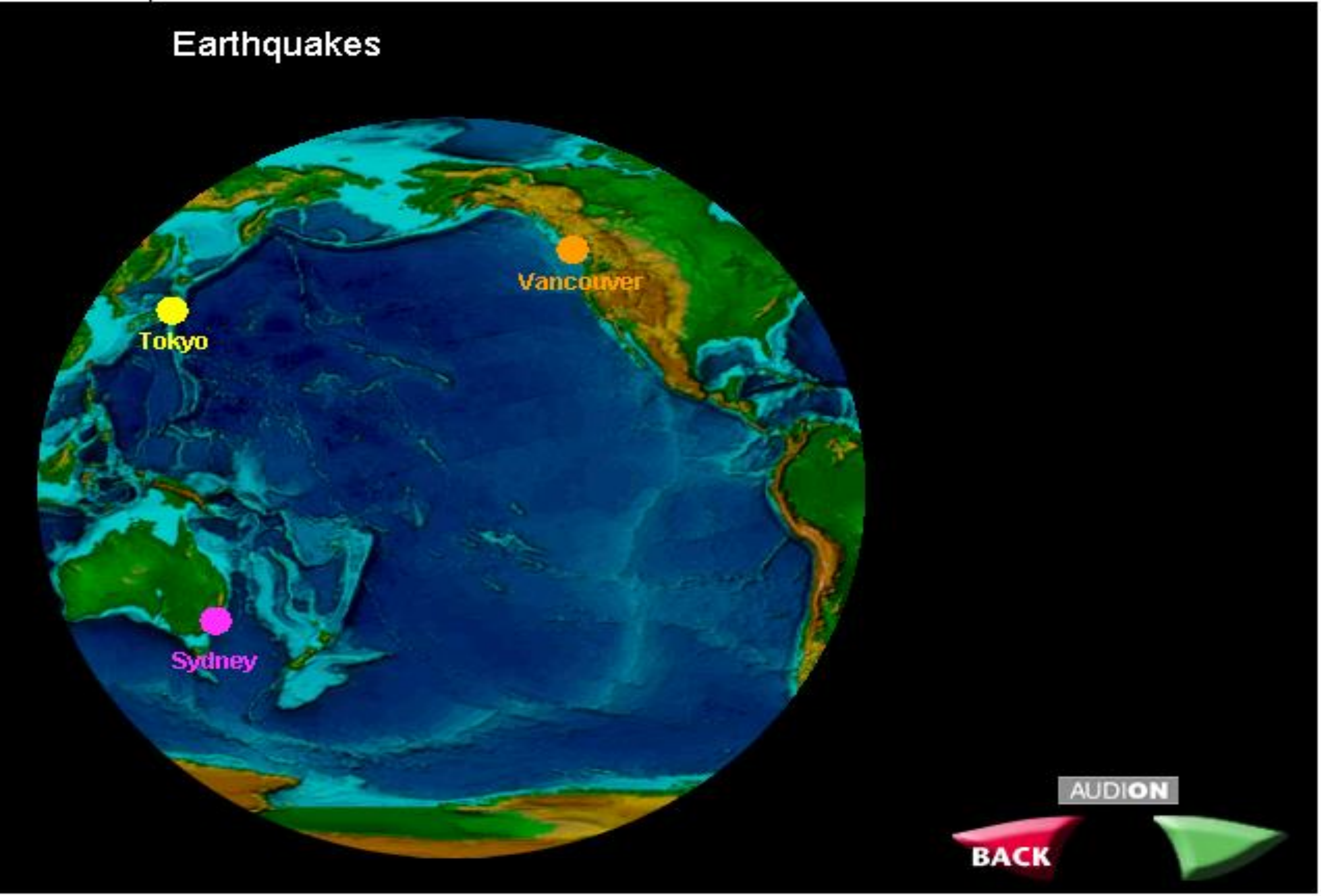
Since each second of interval corresponds to about 8.4 kilometers, we calculate the distance to the epicenter from each of our seismic stations to be:



AUDIO ON

A. Kelly

Courtesy of Dr. Qamar-uz-Zaman Chaudhary
Pakistan Mteorological Dept.



A. Kelly

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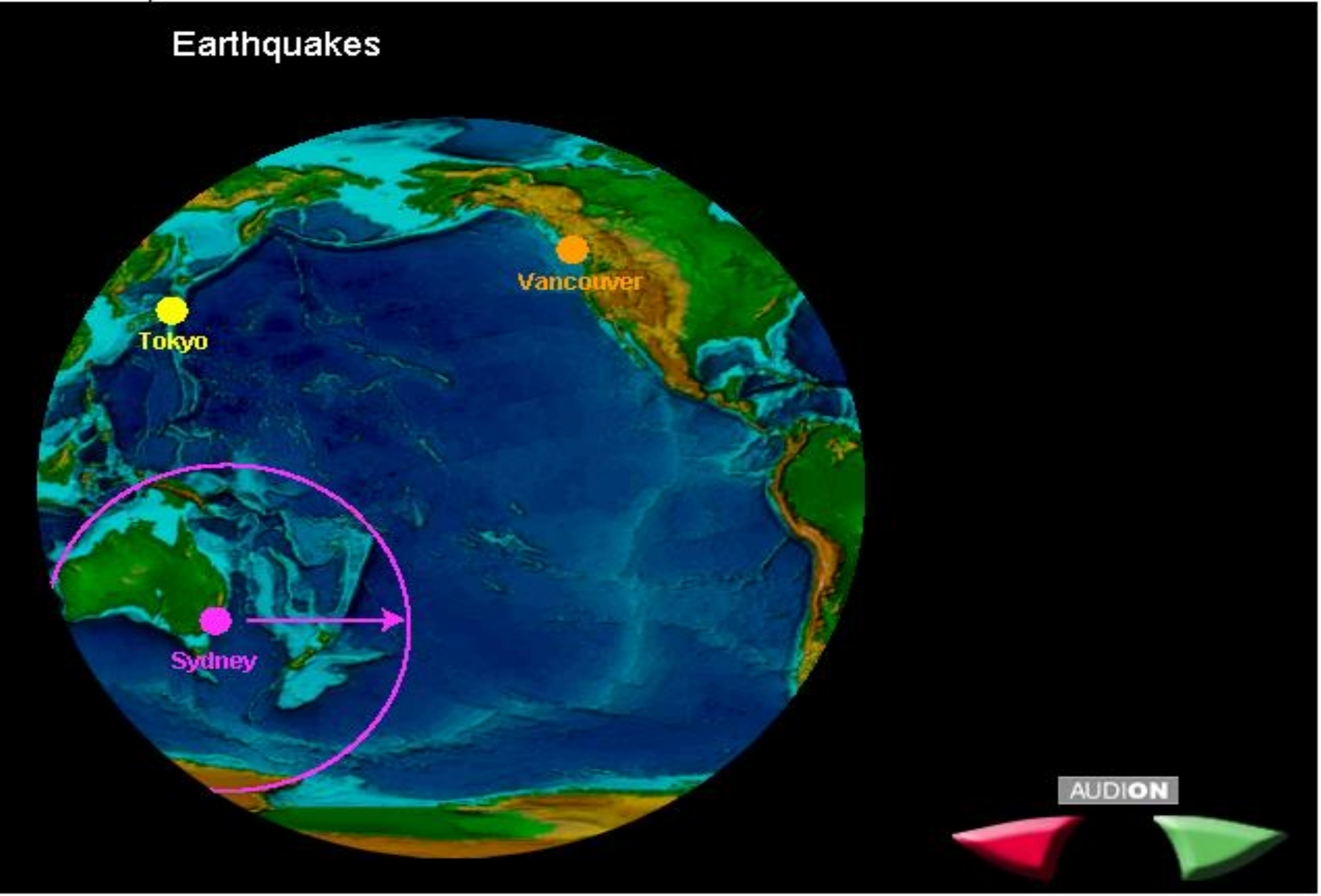

A. Kelly

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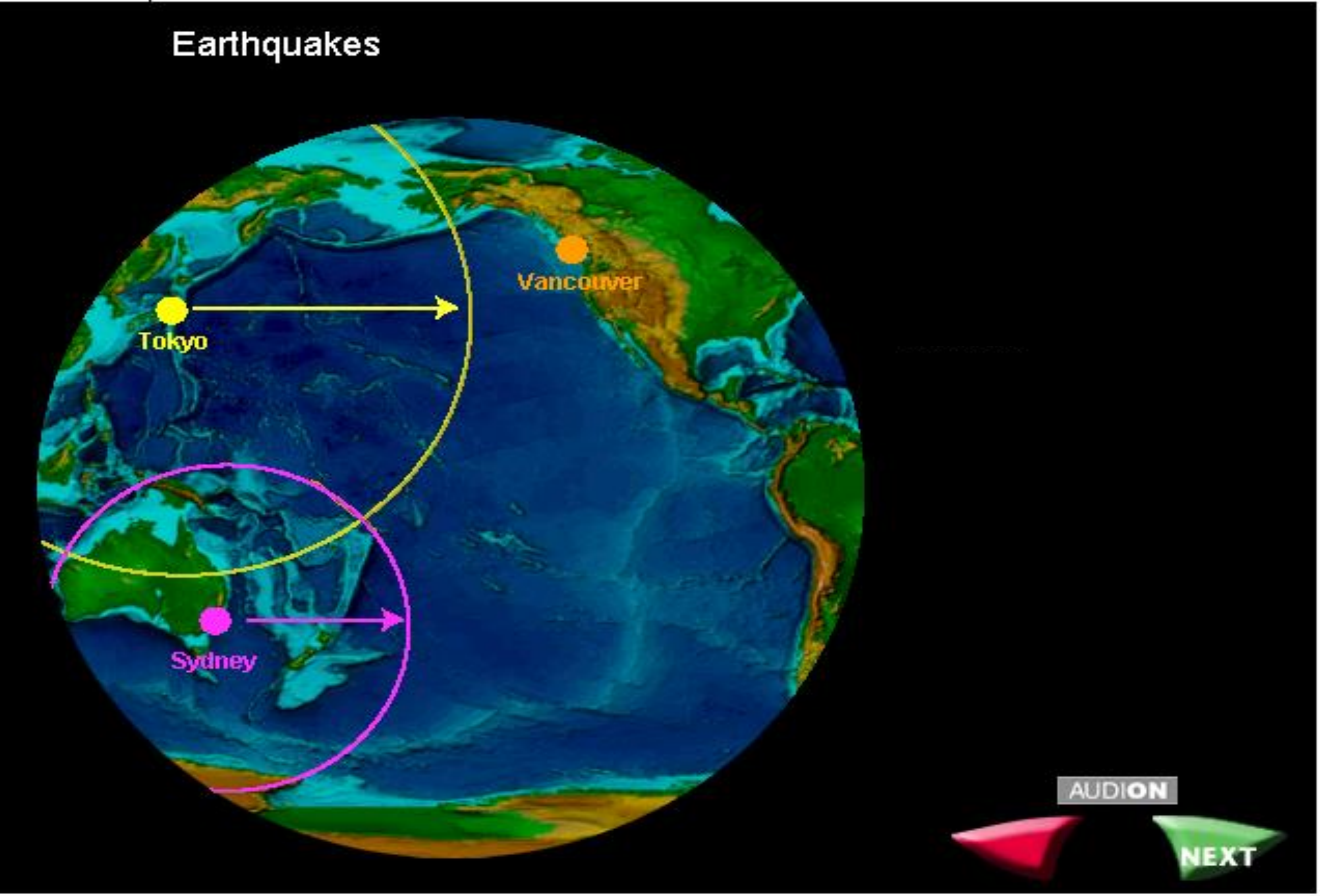

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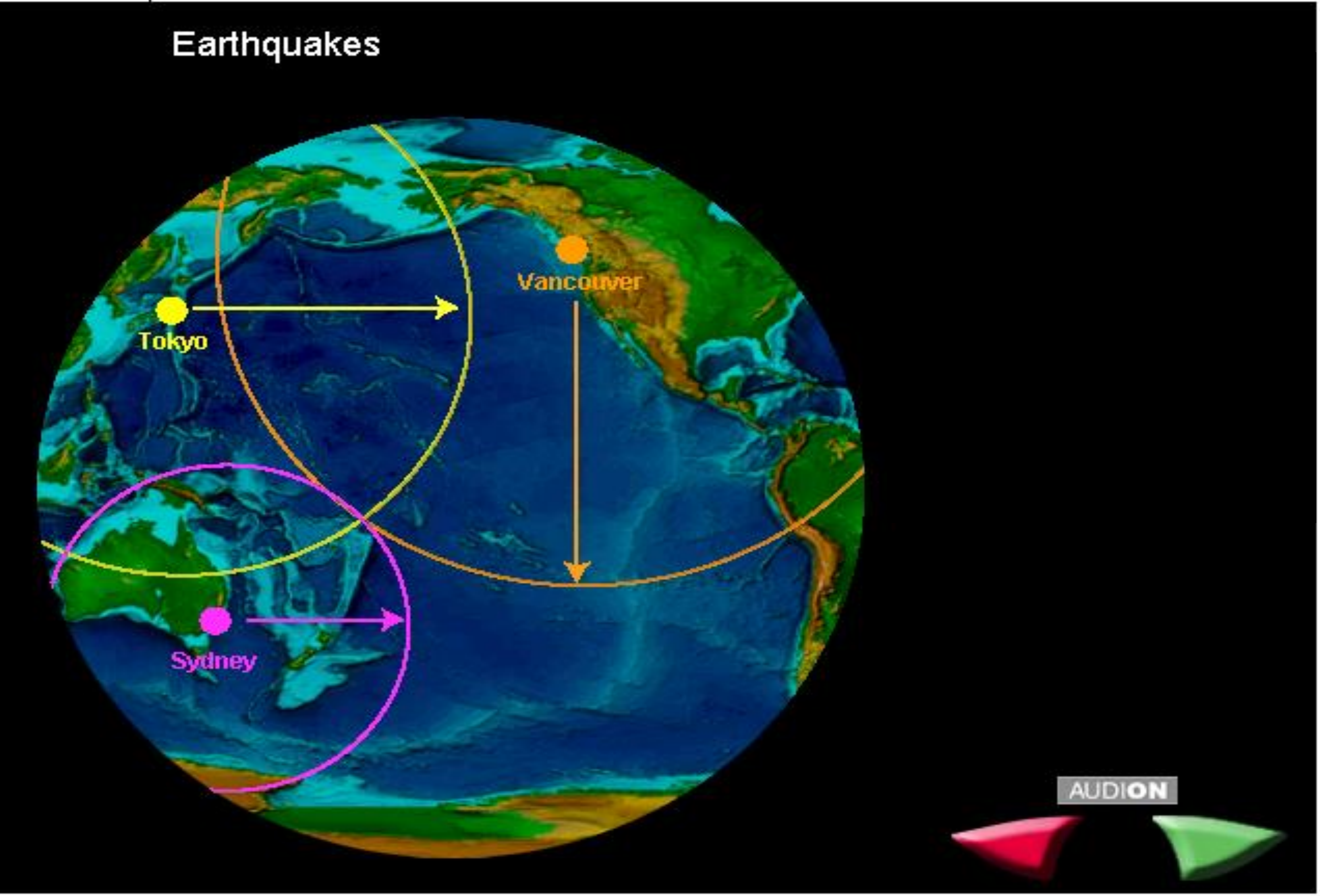
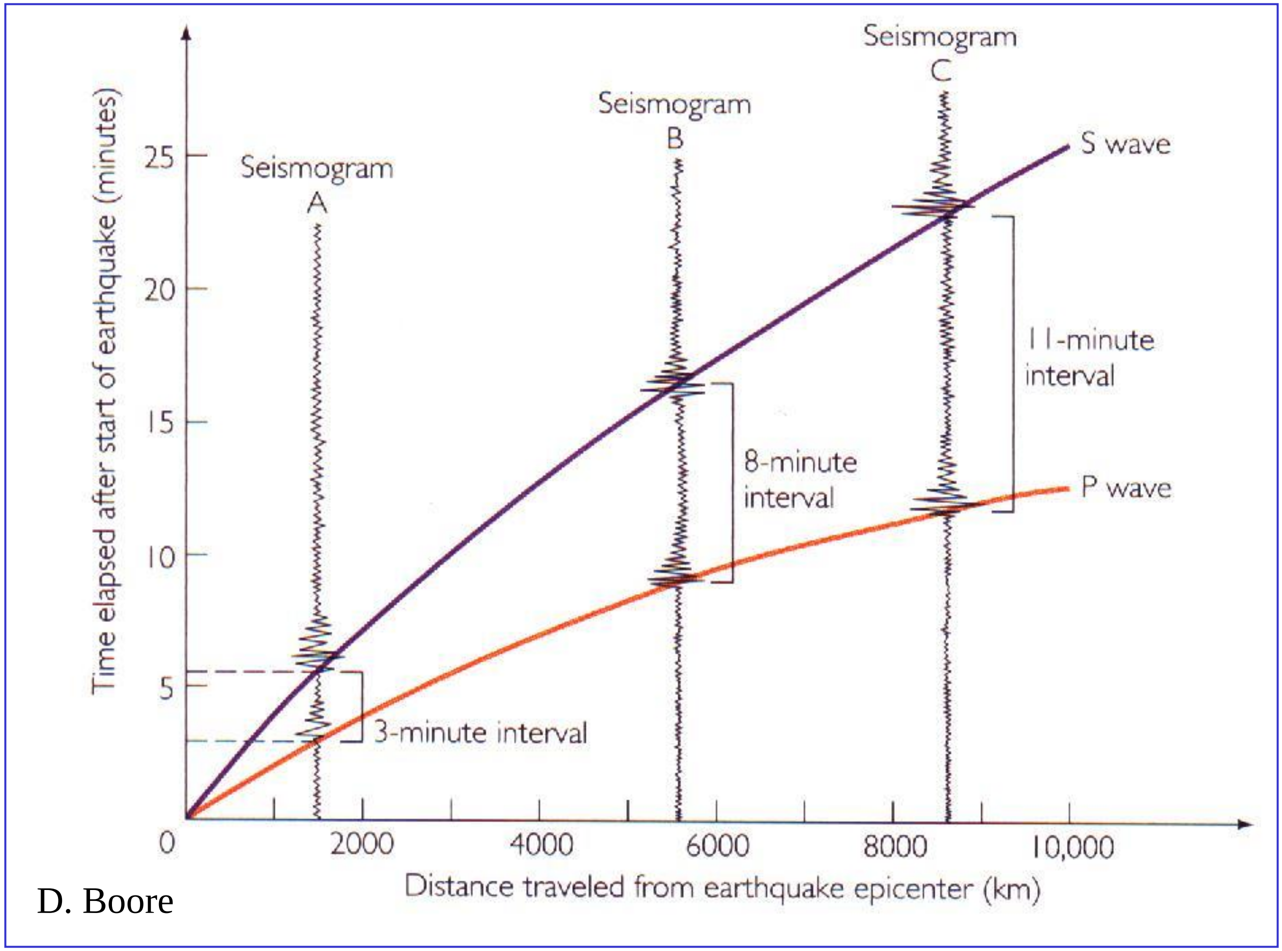


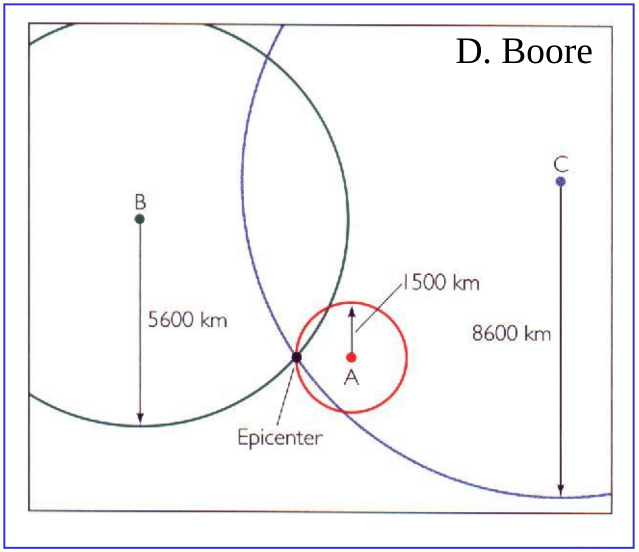
Courtesy of Dr. Qamar-uz-Zaman Chaudhary
Pakistan Mteorological Dept.

A. Kelly

Note: remember that you can use the travel-time curves to estimate the distances..



D. Boore



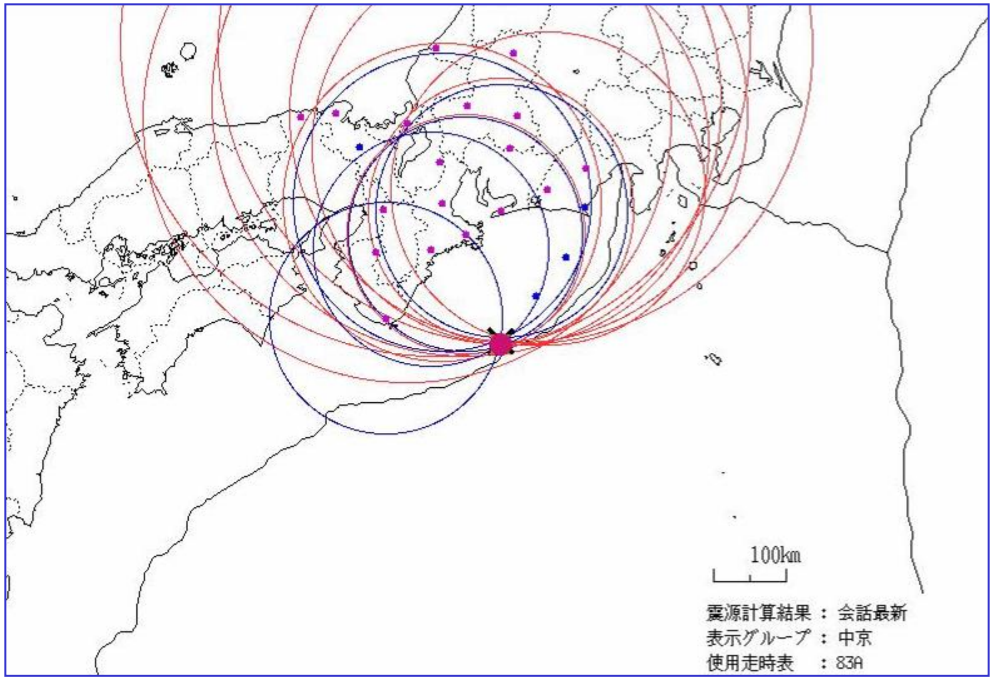
D. Boore

S-P method

- 1 station – know the distance - a circle of possible location
- 2 stations – two circles that will intersect at two locations
- 3 stations – 3 circles, one intersection = unique location

(in absence of errors...)

4+ stations – over determined problem – can get an estimation of errors



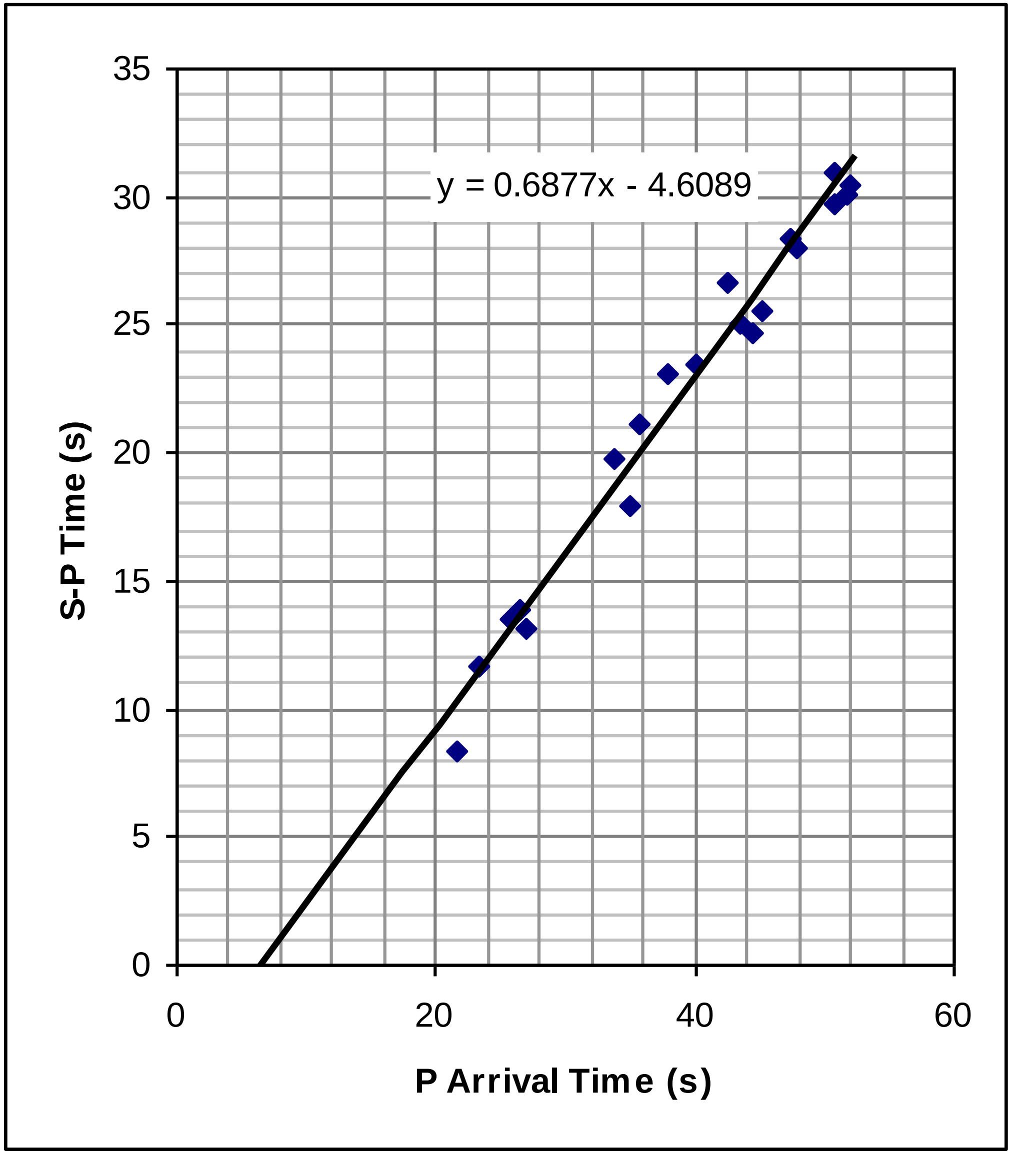
Source: Japan Meteorological Agency

Wadati diagram

S-P time against absolute P arrival time

$$\boxed{t_s - t_p = [v_p / v_s - 1](t_p - t_o)}$$

- gives the origin time (where S-P time = 0)
- Determines Vp/Vs (assuming it's constant and the P and S phases are the same type – e.g. Pn and Sn, or Pg and Sg)
- indicates pick errors



Numerical methods

The arrival time t_i at station i can be written as

$$t_i = \underset{\boxed{\textbf{Travel time}}}{T}(\underbrace{x_i, y_i, z_i}_{\boxed{\text{Station}}}, \underbrace{x_0, y_0, z_0}_{\boxed{\text{Hypocenter}}}) + \underset{\boxed{\text{origin time}}}{t_0} \qquad \text{(n1)}$$

If the arrival times for different stations are known, than the location problem can be solved in a least-squares sense (over-determined system). The minimized quantity is the residual between the observed and the computed arrival times. For station i, the residual is:

$$\boldsymbol{r_i} = t_i^o - t_i^c \qquad \textit{(n2)}$$

Problem: the travel time is a non-linear function of the parameters. For example, in the 2D case:

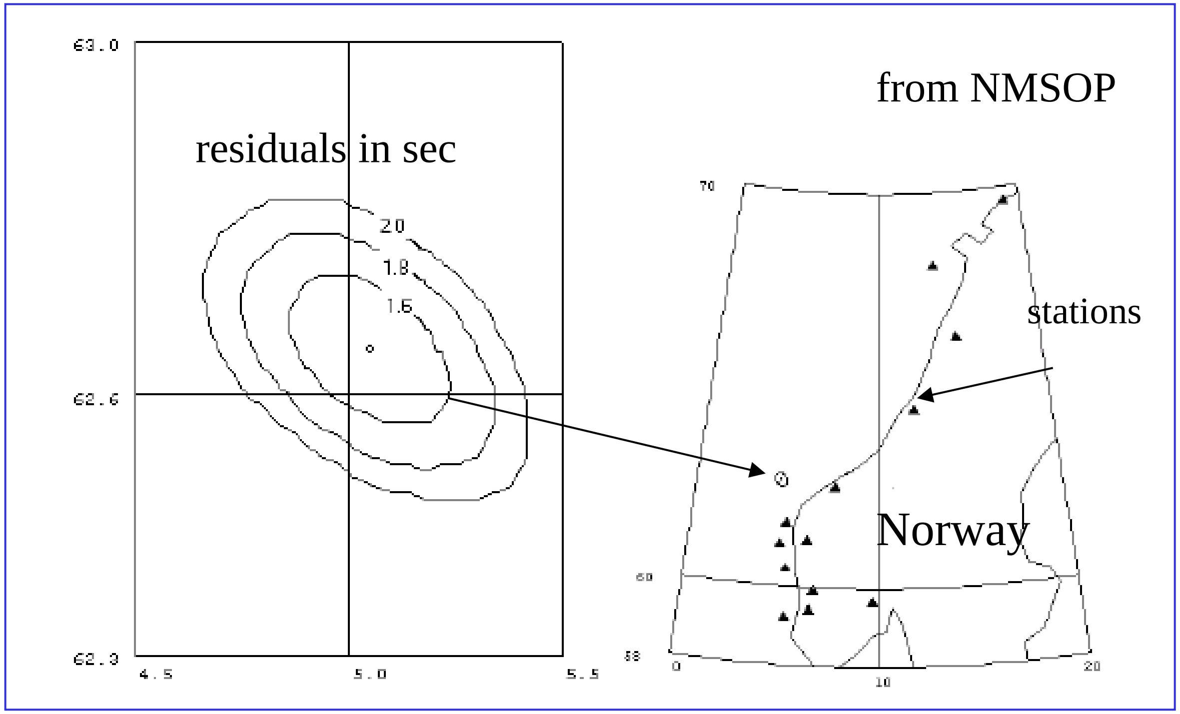
$$\mathrm{T_i} = \frac{\sqrt{(\mathrm{x} - \mathrm{x_i})^2 + (y - y_i)^2}}{v} + t_O \qquad \text{(n3)}$$

Do not forget: the travel time depends on the velocity model!!!

A forward approach can be applied to solve the location problem, e.g. by applying a grid-search scheme. For each considered solution (hypocenter and origin time), the arrival times at each station are computed and the sum of the squared residuals is evaluated:

$$e = \sum_{i=1}^{n} (r_i)^2 \qquad (n4)$$

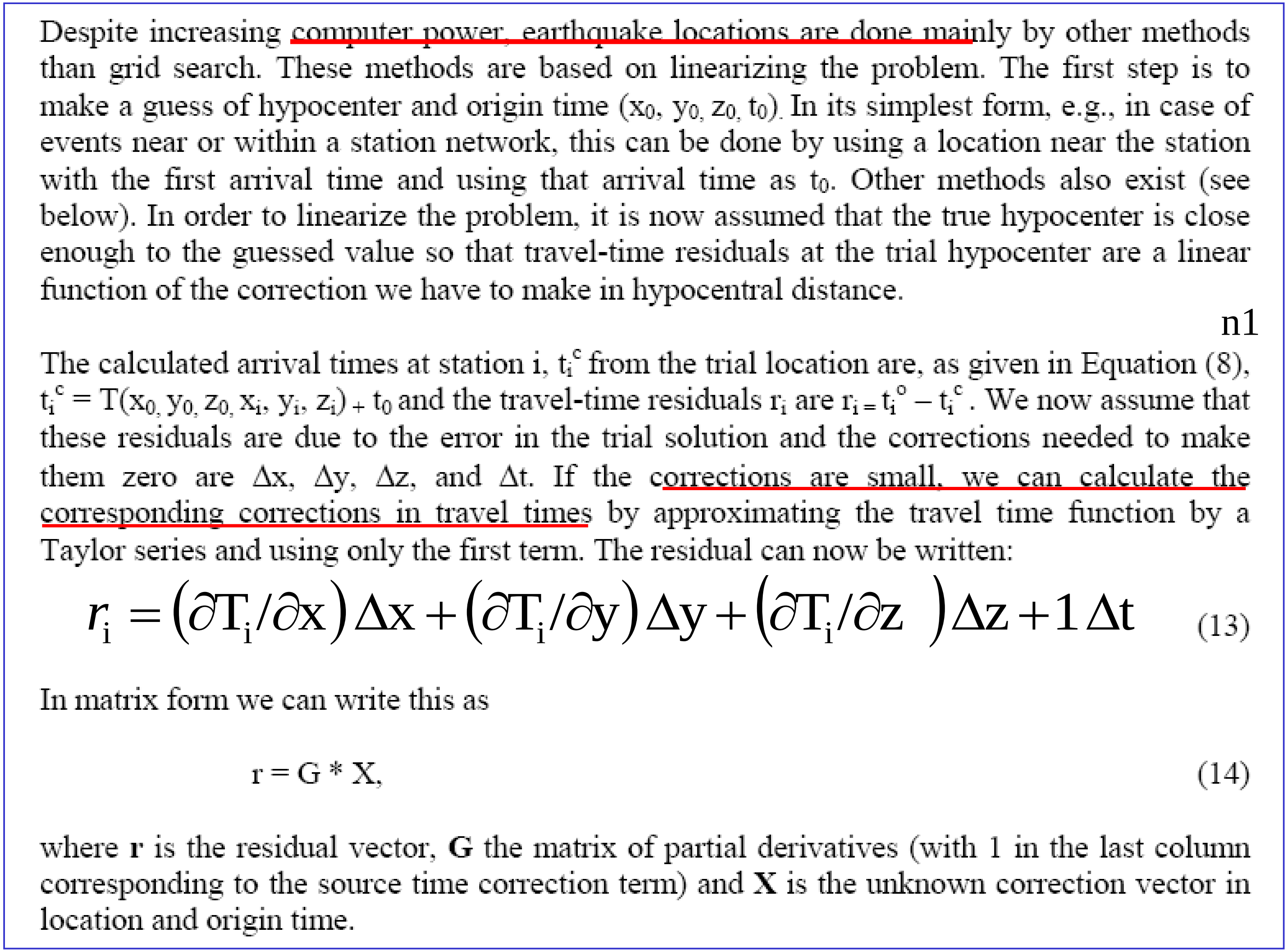
The best solution is that minimize e. The **search strategy** applied for moving in the space of solutions is fundamental for avoiding to select solutions which correspond to local minimum. **This approach is not frequently used** (e.g. Sambridge and Kennett, 1986) but it is useful for investigating whether the solution is constrained or not.



The solution is not constrained in the direction perpendicular to the coast. Can you imagine why?

Iterative methods (Geiger, 1910)

Gieger, L. (1910). Hedbestimmung bie erdbeben aus den ankunftzeiten, *K. Gessel. Wiss. Goett 4*, 331-349.



Despite increasing computer power, earthquake locations are done mainly by other methods than grid search. These methods are based on linearizing the problem. The first step is to make a guess of hypocenter and origin time (x_0, y_0, z_0, t_0). In its simplest form, e.g., in case of events near or within a station network, this can be done by using a location near the station with the first arrival time and using that arrival time as t_0. Other methods also exist (see below). In order to linearize the problem, it is now assumed that the true hypocenter is close enough to the guessed value so that travel-time residuals at the trial hypocenter are a linear function of the correction we have to make in hypocentral distance.

n1

The calculated arrival times at station i, t_i^c from the trial location are, as given in Equation (8), $t_i^c = T(x_{0,}\, y_{0,}\, z_{0,}\, x_i, y_i, z_i)_+\, t_0$ and the travel-time residuals r_i are $r_{i\,=}\, t_i^o - t_i^c$. We now assume that these residuals are due to the error in the trial solution and the corrections needed to make them zero are Δx, Δy, Δz, and Δt. If the corrections are small, we can calculate the corresponding corrections in travel times by approximating the travel time function by a Taylor series and using only the first term. The residual can now be written:

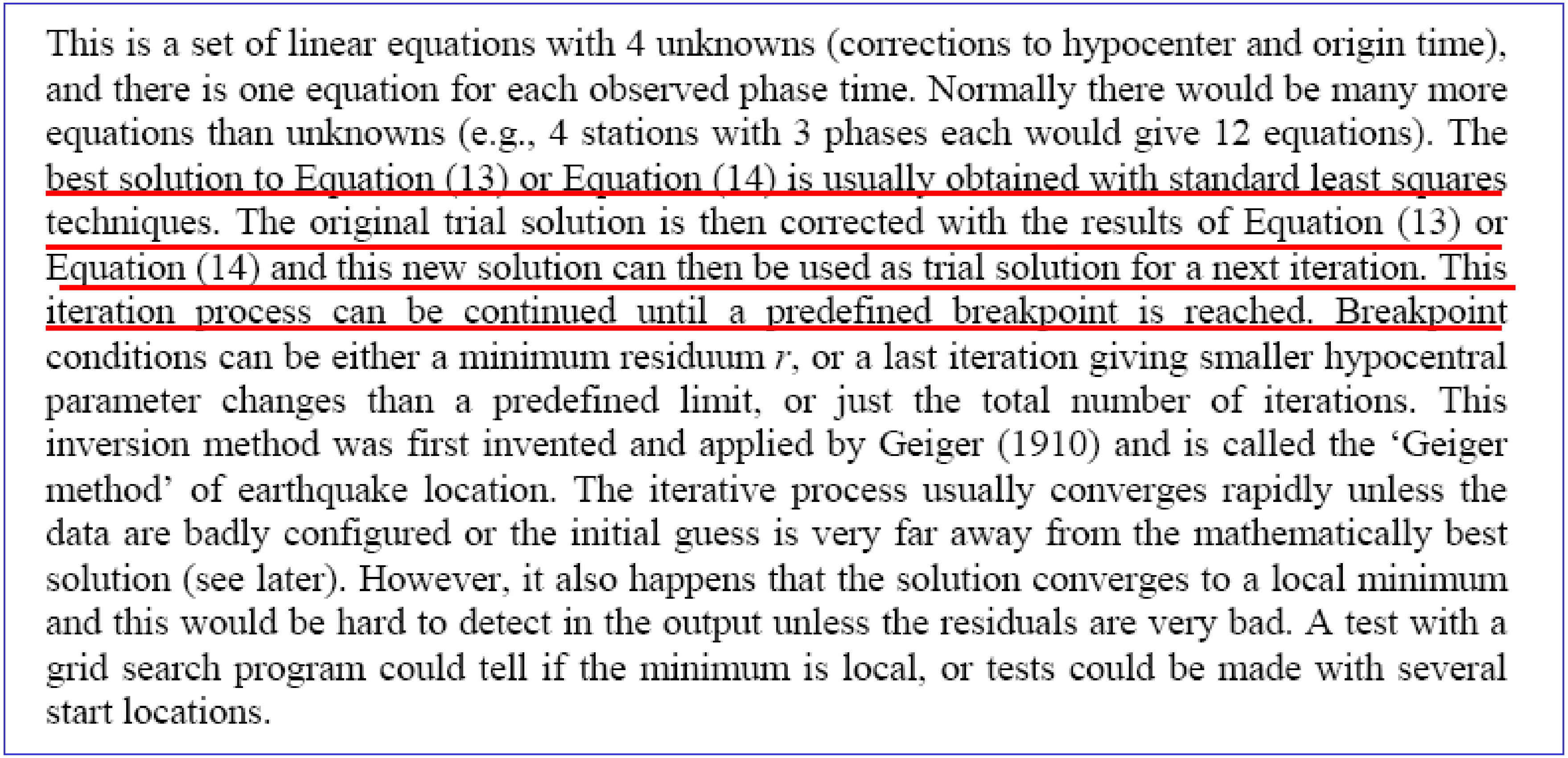
$$r_i = (\partial T_i/\partial x)\Delta x + (\partial T_i/\partial y)\Delta y + (\partial T_i/\partial z\ )\Delta z + 1\,\Delta t \qquad (13)$$

In matrix form we can write this as

$$r = G * X, \qquad (14)$$

where **r** is the residual vector, **G** the matrix of partial derivatives (with 1 in the last column corresponding to the source time correction term) and **X** is the unknown correction vector in location and origin time.

from New Manual of Seismological Observatory Practice



This is a set of linear equations with 4 unknowns (corrections to hypocenter and origin time), and there is one equation for each observed phase time. Normally there would be many more equations than unknowns (e.g., 4 stations with 3 phases each would give 12 equations). The best solution to Equation (13) or Equation (14) is usually obtained with standard least squares techniques. The original trial solution is then corrected with the results of Equation (13) or Equation (14) and this new solution can then be used as trial solution for a next iteration. This iteration process can be continued until a predefined breakpoint is reached. Breakpoint conditions can be either a minimum residuum *r*, or a last iteration giving smaller hypocentral parameter changes than a predefined limit, or just the total number of iterations. This inversion method was first invented and applied by Geiger (1910) and is called the 'Geiger method' of earthquake location. The iterative process usually converges rapidly unless the data are badly configured or the initial guess is very far away from the mathematically best solution (see later). However, it also happens that the solution converges to a local minimum and this would be hard to detect in the output unless the residuals are very bad. A test with a grid search program could tell if the minimum is local, or tests could be made with several start locations.

from New Manual of Seismological Observatory Practice

Earthquake location: theory

1. ` t_i is nonlinear function of t_0, source and station location

$$t_i = t_0 + \tau_i = f(t_0, x_i - x, y_i - y, z_i - z) = f_i(t_0, x, y, z)$$

origin time

travel time at station *i*

2. Linear approximation by Taylor’s series expansion around starting solution ^{0}

$$t_i \approx f_i(t_0^0, x^0, y^0, z^0) + \left.\frac{\partial f_i}{\partial t_0}\right|_0 (t_0 - t_0^0) + \left.\frac{\partial f_i}{\partial x}\right|_0 (x - x^0) + \left.\frac{\partial f_i}{\partial y}\right|_0 (y - y^0) + \left.\frac{\partial f_i}{\partial z}\right|_0 (z - z^0)$$

theoretical arrival for a given location

Earthquake location: inversion approach

$$\begin{pmatrix} t_1 - f_1^0 \\ t_2 - f_2^0 \\ \vdots \\ t_N - f_N^0 \end{pmatrix} = \text{Number of stations} \begin{array}{c} \begin{array}{cccc} \mathit{OT} & \mathit{lat} & \mathit{lon} & \mathit{depth} \end{array} \\ \begin{pmatrix} \frac{\partial f_1}{\partial t_0} & \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial t_0} & \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_N}{\partial t_0} & \frac{\partial f_N}{\partial x} & \frac{\partial f_N}{\partial y} & \frac{\partial f_N}{\partial z} \end{pmatrix} \end{array} \begin{pmatrix} \Delta t_0 \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

Residuals at stations

coefficient matrix
(depend on solution)

step to update model

or

$$\delta \mathrm{t} = \mathbf{G} \boldsymbol{\Delta} \mathbf{m}$$

linear system: solved by least squares

T. Dahm

Least Squares Inversion (linearized approach)

From $\delta \mathbf{t} = \mathbf{G} \boldsymbol{\Delta} \mathbf{m}$

a least squares solution is constructed as

$$\boldsymbol{\Delta} \mathbf{m} = \mathbf{G}^{-1} \delta \mathbf{t}$$

After 1 iteration the improved location is

$$t_0 = t_0^0 + \Delta t_0, \quad x = x^0 + \Delta x, \quad y = y^0 + \Delta y, \quad z = z^0 + \Delta z$$

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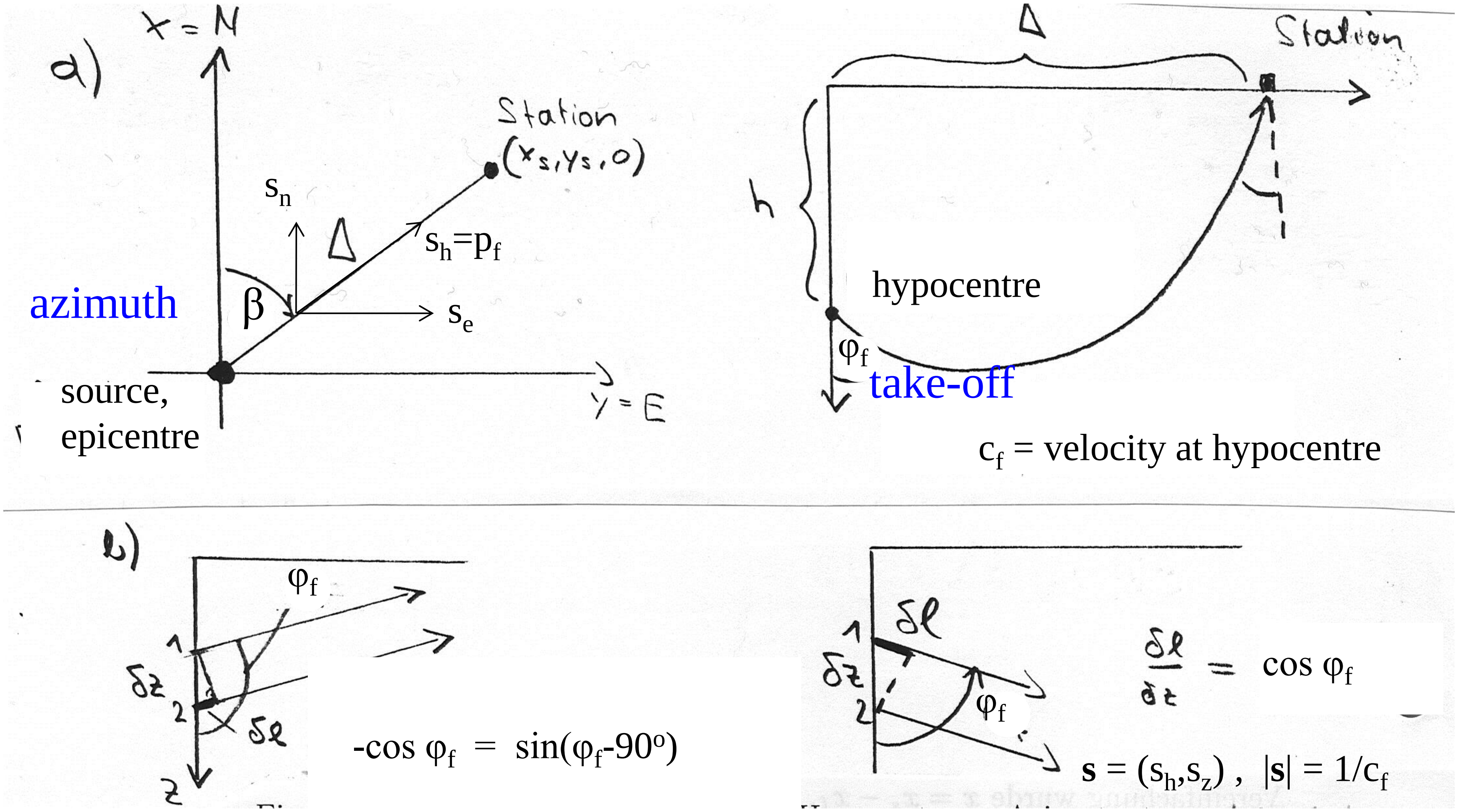
Sketch on ray geometry



c_f = velocity at hypocentre

$$-\cos\varphi_f = \sin(\varphi_f - 90^o)$$

$$\frac{\delta l}{\delta z} = \cos\varphi_f$$

$$\mathbf{s} = (s_h, s_z)\ ,\ \ |\mathbf{s}| = 1/c_f$$

slowness vector at source,
shows direction of ray

T. Dahm

Layered Earth: partial derivatives can be equated

$$\boxed{\frac{\partial f}{\partial t_0}} = \frac{\partial(t_0+\tau)}{\partial t_0} = 1$$

(in general)

Take-off

$$\boxed{\frac{\partial f}{\partial x}} = \frac{\partial f}{\partial \Delta}\frac{\partial \Delta}{\partial x} = \frac{\sin\varphi_f}{c_f}\cos\beta = p_f\cos\beta = s_n$$

azimuth

*s_n is the north component of the slowness vector **s**,*
p_f is the ray parameter in a flat earth model (= horizontal slowness)

$$\boxed{\frac{\partial f}{\partial y}} = \frac{\partial f}{\partial \Delta}\frac{\partial \Delta}{\partial y} = \frac{\sin\varphi_f}{c_f}\sin\beta = p_f\sin\beta = s_e$$

$$\boxed{\frac{\partial f}{\partial z}} = \lim\frac{f_2-f_1}{z_2-z_1} = \frac{\delta l/c_f}{\delta z} \approx \pm\frac{\cos\varphi_f}{c_f} = \pm\sqrt{\frac{1}{c_f^2}-p_f^2} = s_z$$

*s_z is the vertical component of the slowness vector **s***

T. Dahm

$$\begin{pmatrix} t_1 - f_1^0 \\ t_2 - f_2^0 \\ \vdots \\ t_N - f_N^0 \end{pmatrix} = \begin{pmatrix} \frac{\partial f_1}{\partial t_0} & \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial t_0} & \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial f_N}{\partial t_0} & \frac{\partial f_N}{\partial x} & \frac{\partial f_N}{\partial y} & \frac{\partial f_N}{\partial z} \end{pmatrix} \begin{pmatrix} \Delta t_0 \\ \Delta x \\ \Delta y \\ \Delta z \end{pmatrix}$$

always 1

s_z of each ray

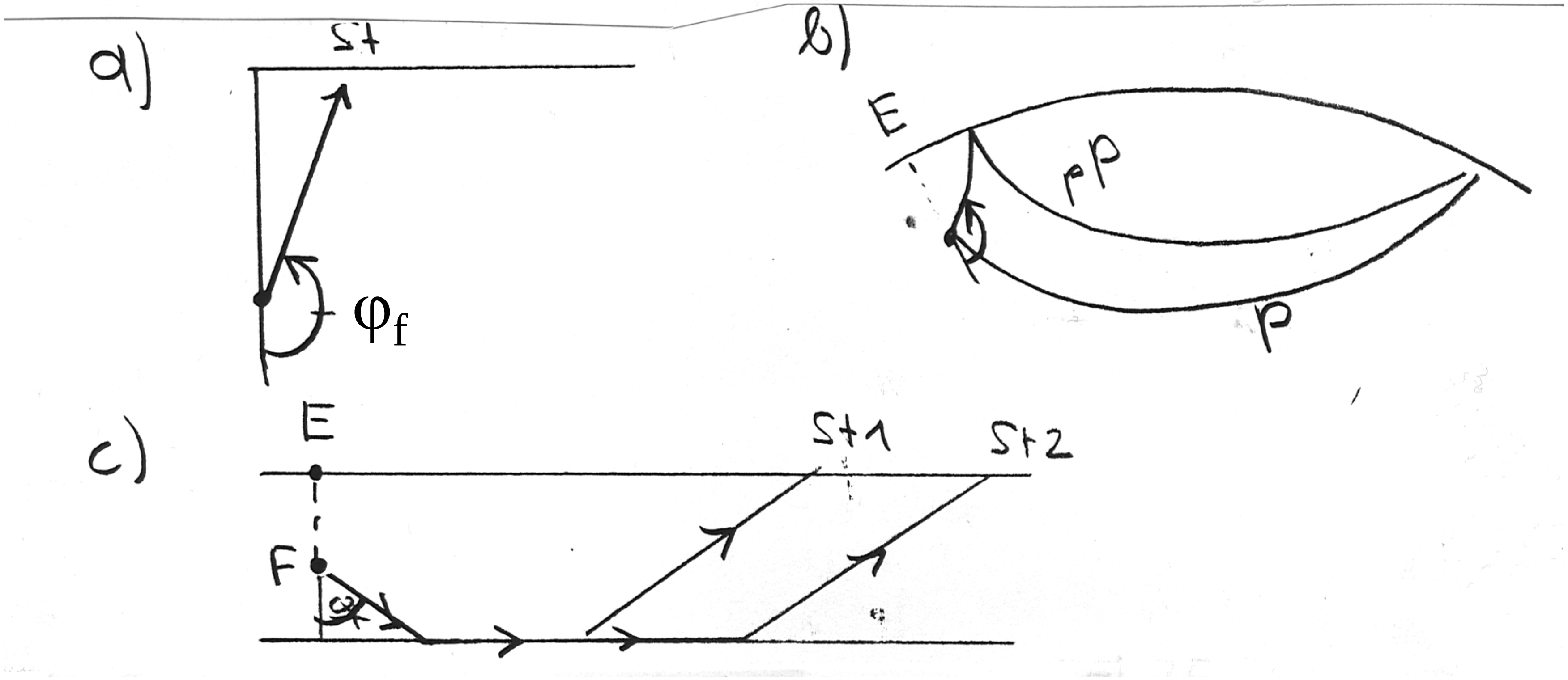
✓ at least 4 independent picks needed to locate an earthquake (much more recommended!)

✓ large partial derivatives give higher weight to associated source parameter

✓ If column vectors in coefficient matrix have similar ‘shape’ the associated source parameter cannot be independently be resolved

T. Dahm

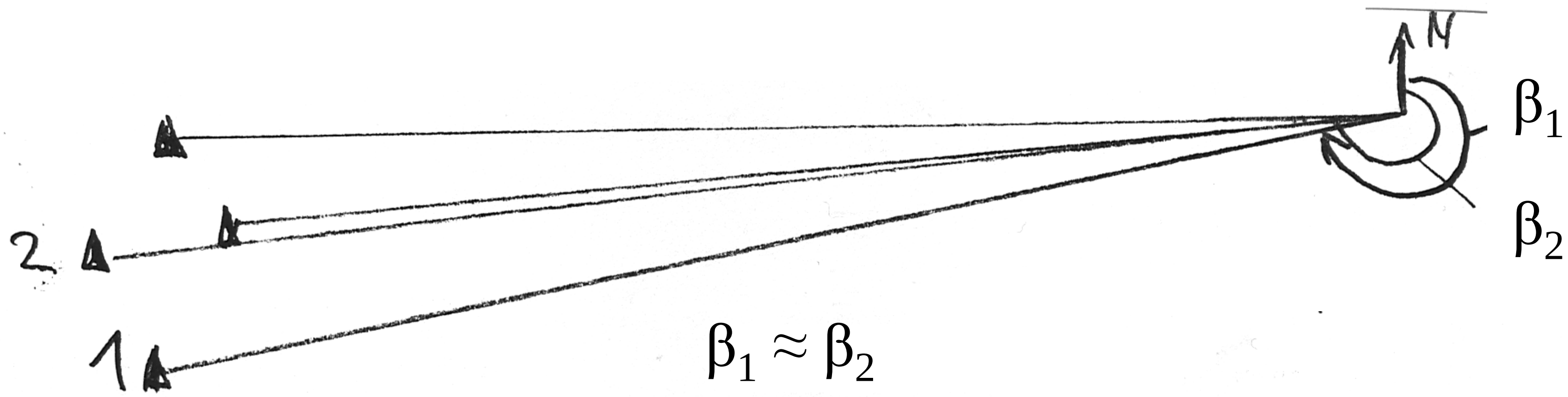
(1a) Depth resolved best if epicentral stations and/or depth phases used (e.g. pP) ($s_z = \cos\varphi_f / c_f$)



(1b) using only P_n phases cannot resolve depth independent from source time (1^{st} + last column constant)

T. Dahm

(2) Epicentres from EQ outside the network have large uncertainties (no variation of s_e and s_n)



the azimuthal gap (largest azimuth angle between adjacent rays) is therefore a criterion of the goodness of location

T. Dahm

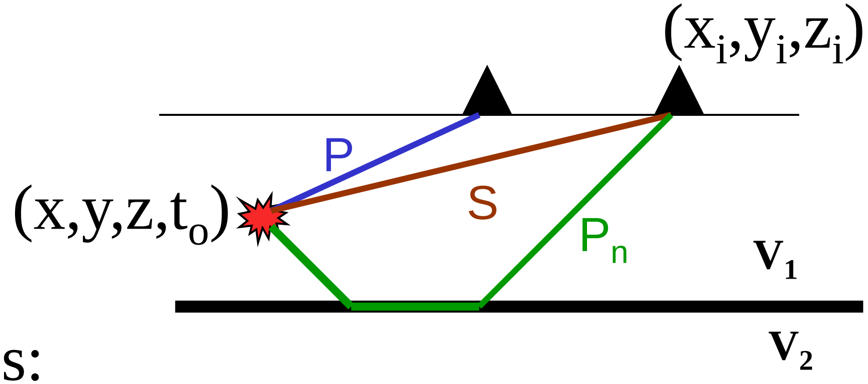
(3) S-phases additionally to P reduce trade-off between source depth and source time

... since $s_z = \cos\varphi_f / c_f$,
and c_f *varies for P and S waves*

The seismologist has to find a compromise between uncertain S picks and breaking off linear dependencies in coefficient matrix.

A recommendation is to use at least 1 S-phase additional to P

Example: homogeneous crust



Arrival time for direct waves is:

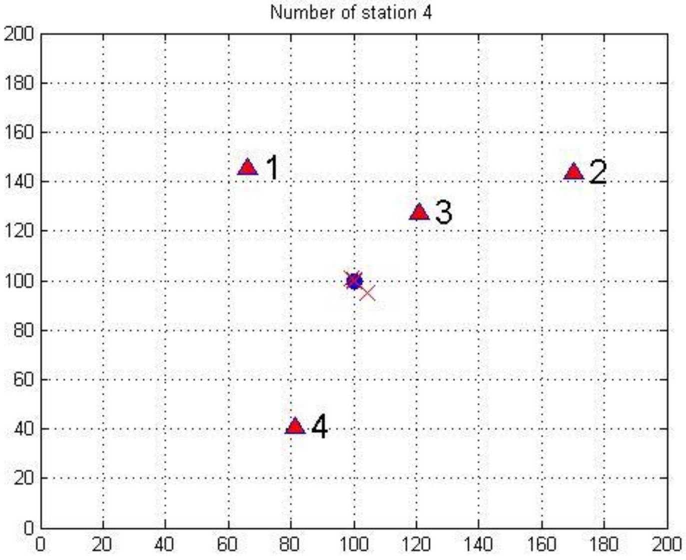
$$T_i = \frac{\sqrt{(x - x_i)^2 + (\boldsymbol{y} - \boldsymbol{y_i})^2 + (\boldsymbol{z} - \boldsymbol{z_i})^2}}{\boldsymbol{v}} + t_0 \qquad (n6)$$

where $V=V_1$. The partial derivatives are (e.g. for x)

$$\partial T_i / \partial \boldsymbol{x} = \frac{(\boldsymbol{x} - \boldsymbol{x_i})}{\boldsymbol{v}} \frac{1}{\sqrt{(x - x_i)^2 + \cdots}} \qquad (n7)$$

Similar expressions can be made for y and z.

Only P

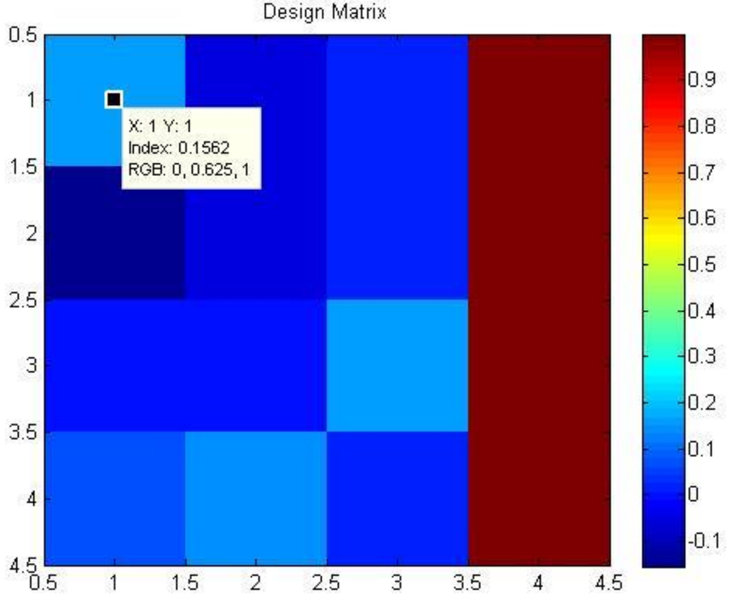


station1

station2

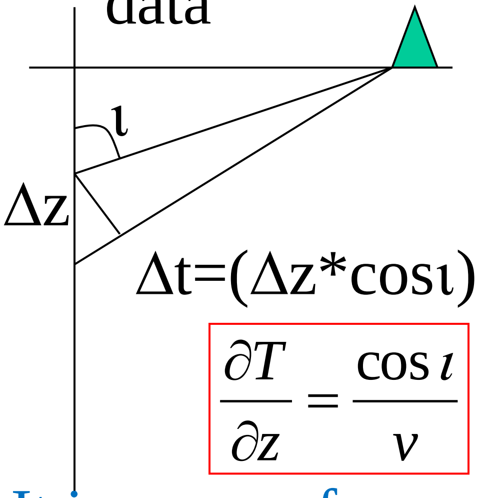
station3

station4



$\frac{\partial T}{\partial X}$ $\frac{\partial T}{\partial Y}$ $\frac{\partial T}{\partial Z}$ $\frac{\partial T}{\partial to}$

same number of unknowns and data



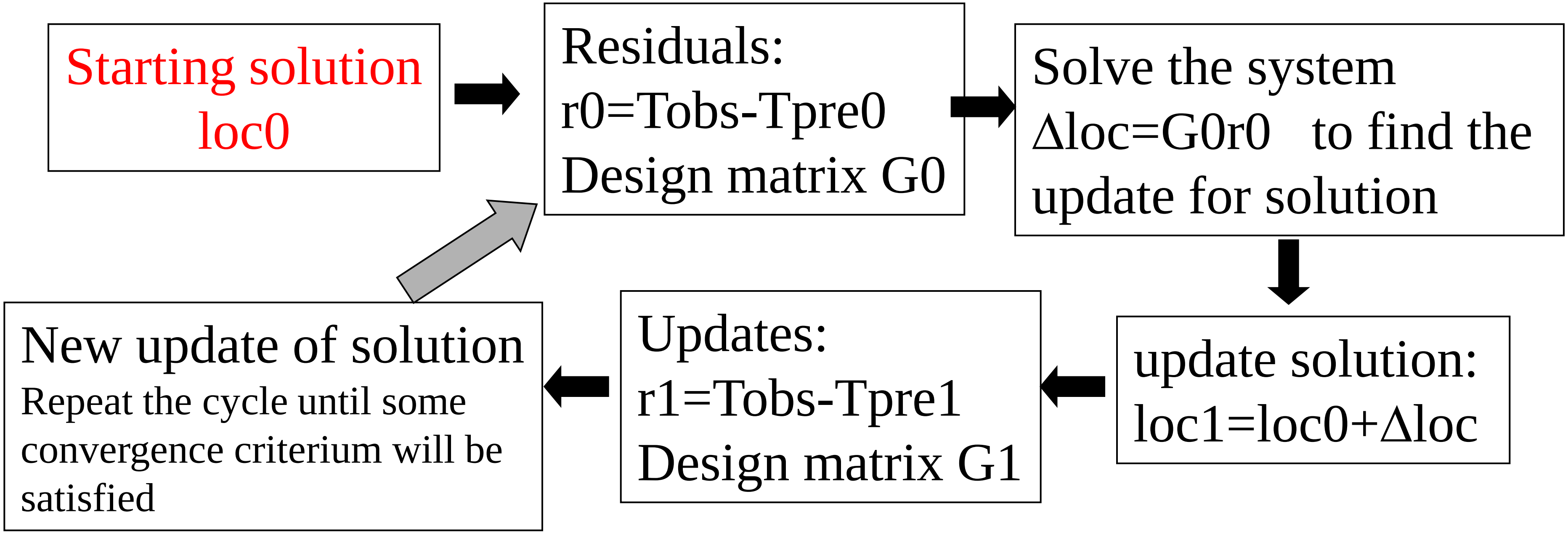
$$r_{\mathrm{i}} = \left(\partial \mathrm{T_i}/\partial \mathrm{x}\right)\Delta \mathrm{x} + \left(\partial \mathrm{T_i}/\partial \mathrm{y}\right)\Delta \mathrm{y} + \left(\partial \mathrm{T_i}/\partial \mathrm{z}\ \right)\Delta \mathrm{z} + \Delta \mathrm{t}$$

Δt=(Δz*cosι)/v

$$\boxed{\frac{\partial T}{\partial z} = \frac{\cos \iota}{v}}$$

$$\partial T_i / \partial \mathbf{x} = \frac{(x - x_i)}{v} \frac{1}{\sqrt{(x - x_i)^2 + \cdots}} \qquad (n7)$$

It increases for small take-off angles, then for stations close to epicentre



Starting solution loc0

Residuals:
r0=Tobs-Tpre0
Design matrix G0

Solve the system Δloc=G0r0 to find the update for solution

update solution:
loc1=loc0+Δloc

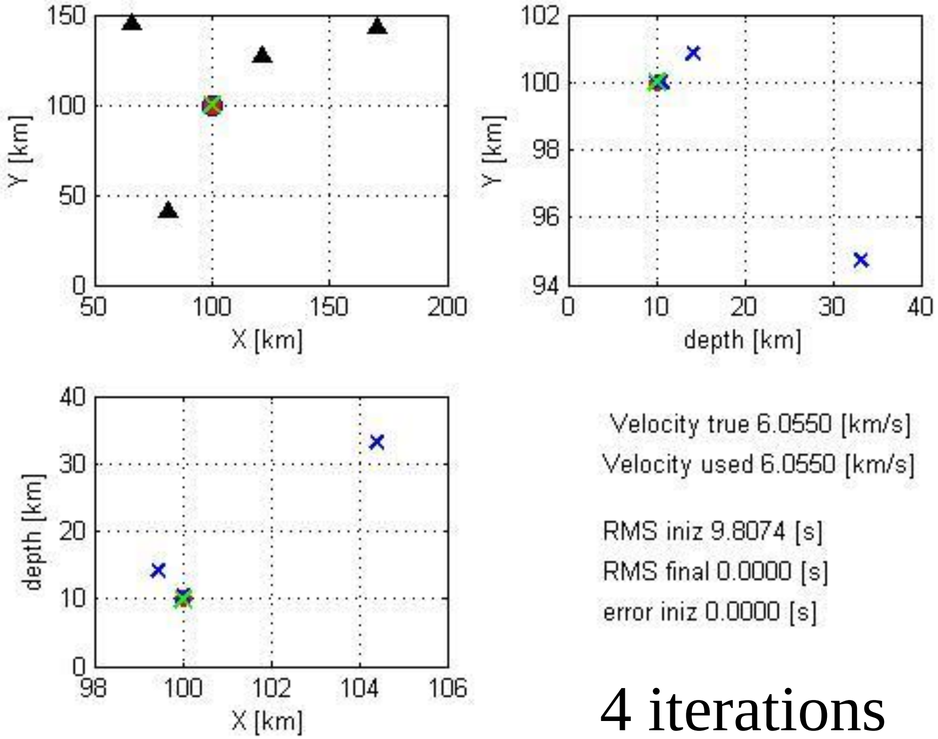
Updates:
r1=Tobs-Tpre1
Design matrix G1

New update of solution
Repeat the cycle until some convergence criterium will be satisfied

Noise free data (and exact velocity model)

Starting solution:
[120.97, 127.19,5,10] km

(generally x,y of station having minimum travel time)



Velocity true 6.0550 [km/s]
Velocity used 6.0550 [km/s]

RMS iniz 9.8074 [s]
RMS final 0.0000 [s]
error iniz 0.0000 [s]

4 iterations

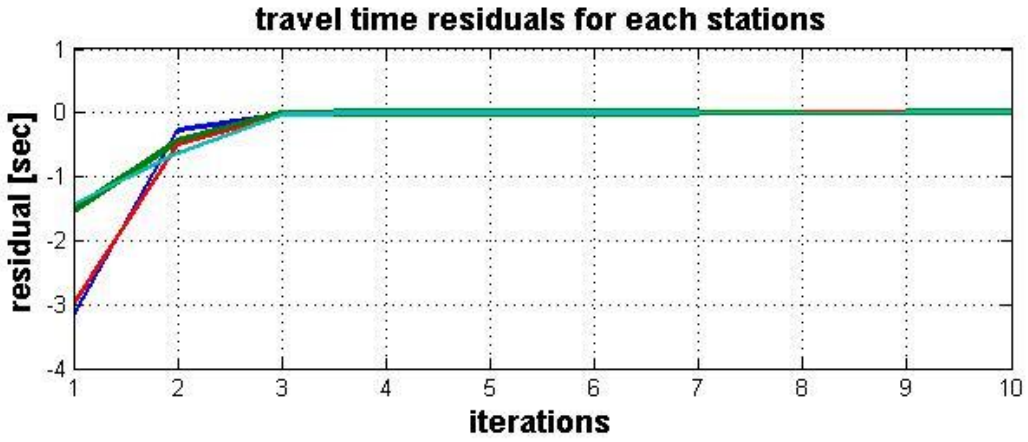
noisy corrupted data: approximate solution

The measured arrival times are affected by errors (e.g. picking errors, systematic time off-set errors).
Also the velocity model is just a simplification of true Earth properties (observed and theoretical predicted arrival-times cannot be the same).

it is generally assumed that the errors have Gaussian distribution and that there are no systematic errors like clock error. It is also assumed that there are no errors in the theoretical travel times, back-azimuths, or ray parameter calculations due to unknown structures. This is of course not true in real life, however error calculations become too difficult if we do not assume a simple error distribution.

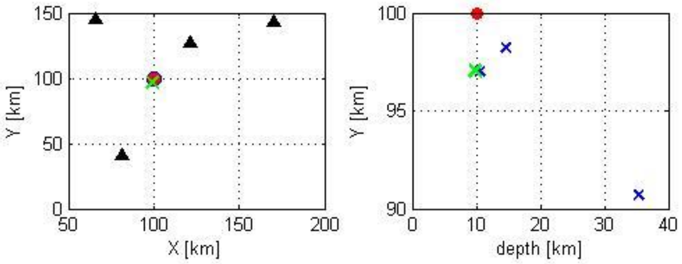
noisy corrupted data: approximate solution

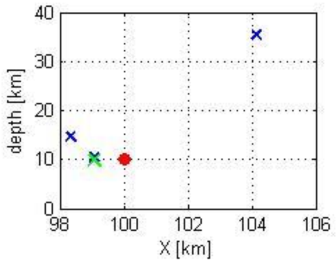
Gaussian noise with zero mean and 0.2sec of stdev



True Model:(100.000 100.000 10.000 [km]; 0.000 [sec])

Final Model:(99.068 97.046 9.785 [km]; 0.015 [sec])





Velocity true 6.0550 [km/s]
Velocity used 6.0550 [km/s]

RMS iniz 9.8219 [s]
RMS final 0.0000 [s]
error iniz 0.4255 [s]

10 iterations

noisy corrupted data: least-squares solution

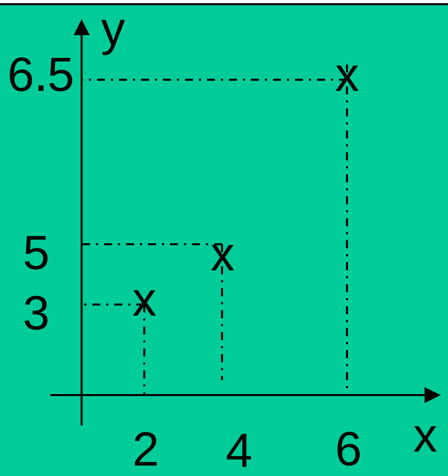
To mitigate the bias due to errors, more than 4 phases are generally considered (at least 7, 10, or even more). Then, we have more data than unknowns → over-determined system.

Datum (x3,y3)=(6,6.5)

$$\begin{bmatrix} 1 & 2 \\ 1 & 4 \\ 1 & 6 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 6.5 \end{bmatrix}$$

It has NO solution

(overdeterimined problem)



The solution is…..to generalize the concept of solution!!!!
We consider “**approximate**” solutions.

LEAST SQUARES SOLUTION

Gauss 1795

ORIGINAL PROBLEM

$Ax=b$

LEAST SQUARES SOLUTION:

$A^TAx=A^Tb$

$$\begin{bmatrix} 1 & 2 \\ 1 & 4 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \\ 6.5 \end{bmatrix}$$

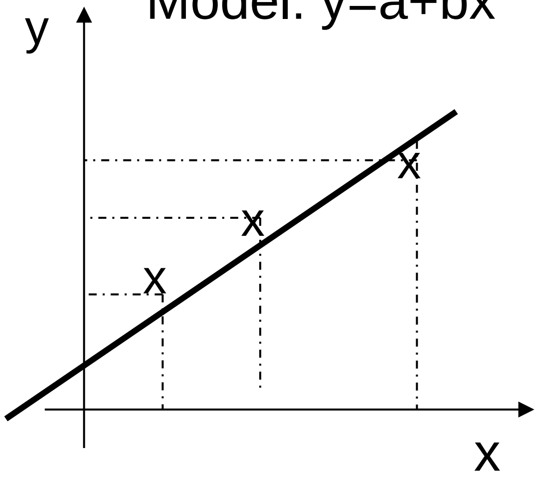
Normal equation (or Euler equation)

$A^tAx=A^ty$ →

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 1 & 4 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 4 & 6 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \\ 6.5 \end{bmatrix}$$

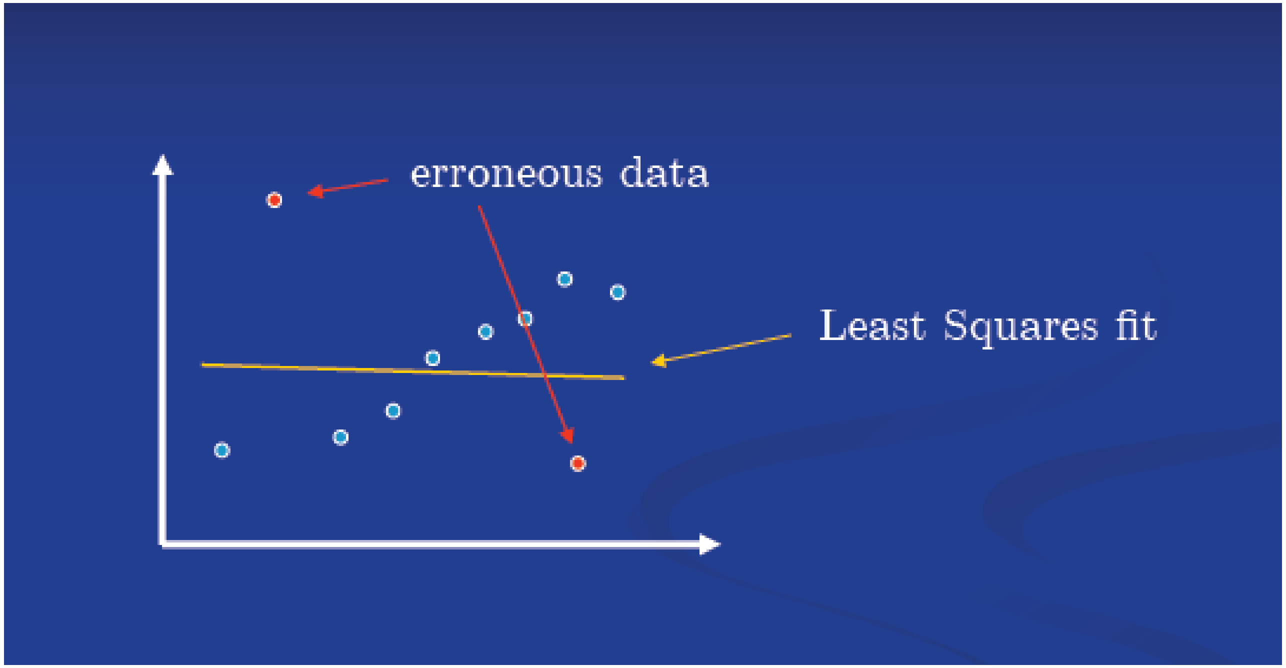
$$\begin{bmatrix} 3 & 12 \\ 12 & 56 \end{bmatrix}\begin{bmatrix} \mathrm{a} \\ \mathrm{b} \end{bmatrix} = \begin{bmatrix} 14.5 \\ 65 \end{bmatrix} \rightarrow \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 4/3 \\ 7/8 \end{bmatrix}$$

Model: y=a+bx

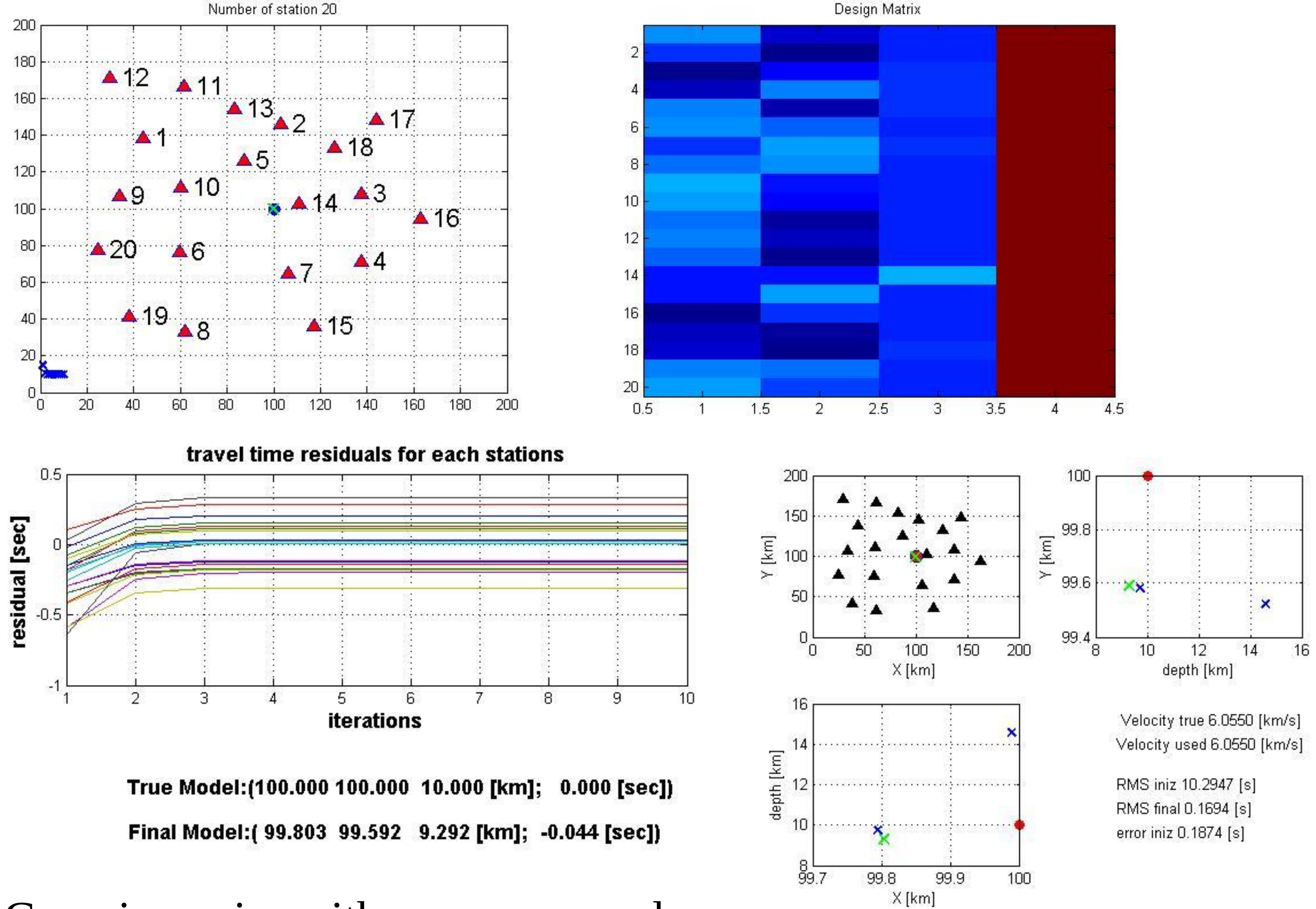


It is the solution that minimize the 2-norm of $||Ax-y||_2$

where $$\|x\|_2 = \sqrt{x_1^2 + x_2^3 + x_3^2 + \cdots + x_n^2}$$



With least squares be careful to outliers !!!!!!!!



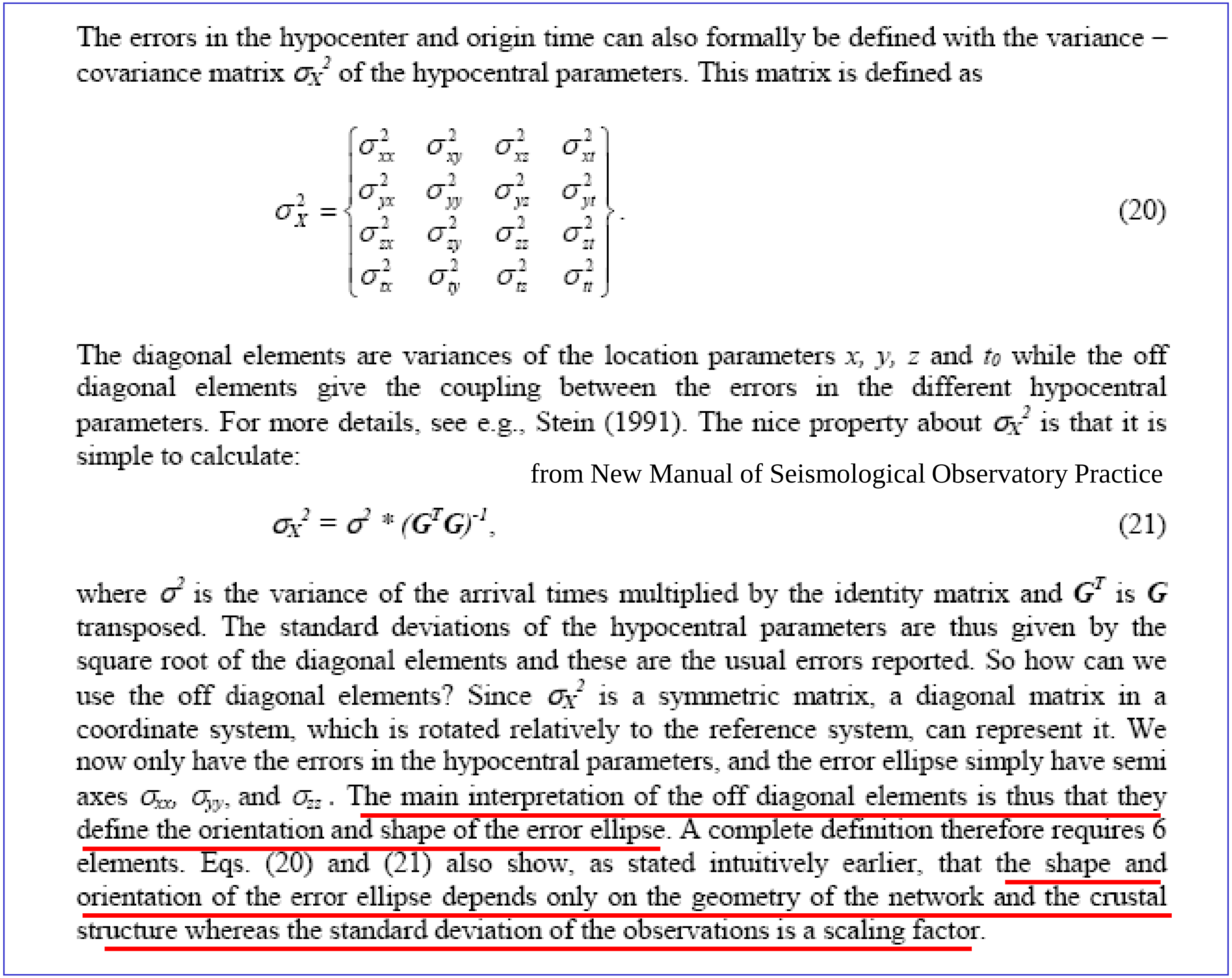
True Model:(100.000 100.000 10.000 [km];   0.000 [sec])

Final Model:(99.803 99.592   9.292 [km]; -0.044 [sec])

Velocity true 6.0550 [km/s]
Velocity used 6.0550 [km/s]

RMS iniz 10.2947 [s]
RMS final 0.1694 [s]
error iniz 0.1874 [s]

Gaussian noise with zero mean and 0.2sec of standard deviation



The errors in the hypocenter and origin time can also formally be defined with the variance – covariance matrix σ_X^2 of the hypocentral parameters. This matrix is defined as

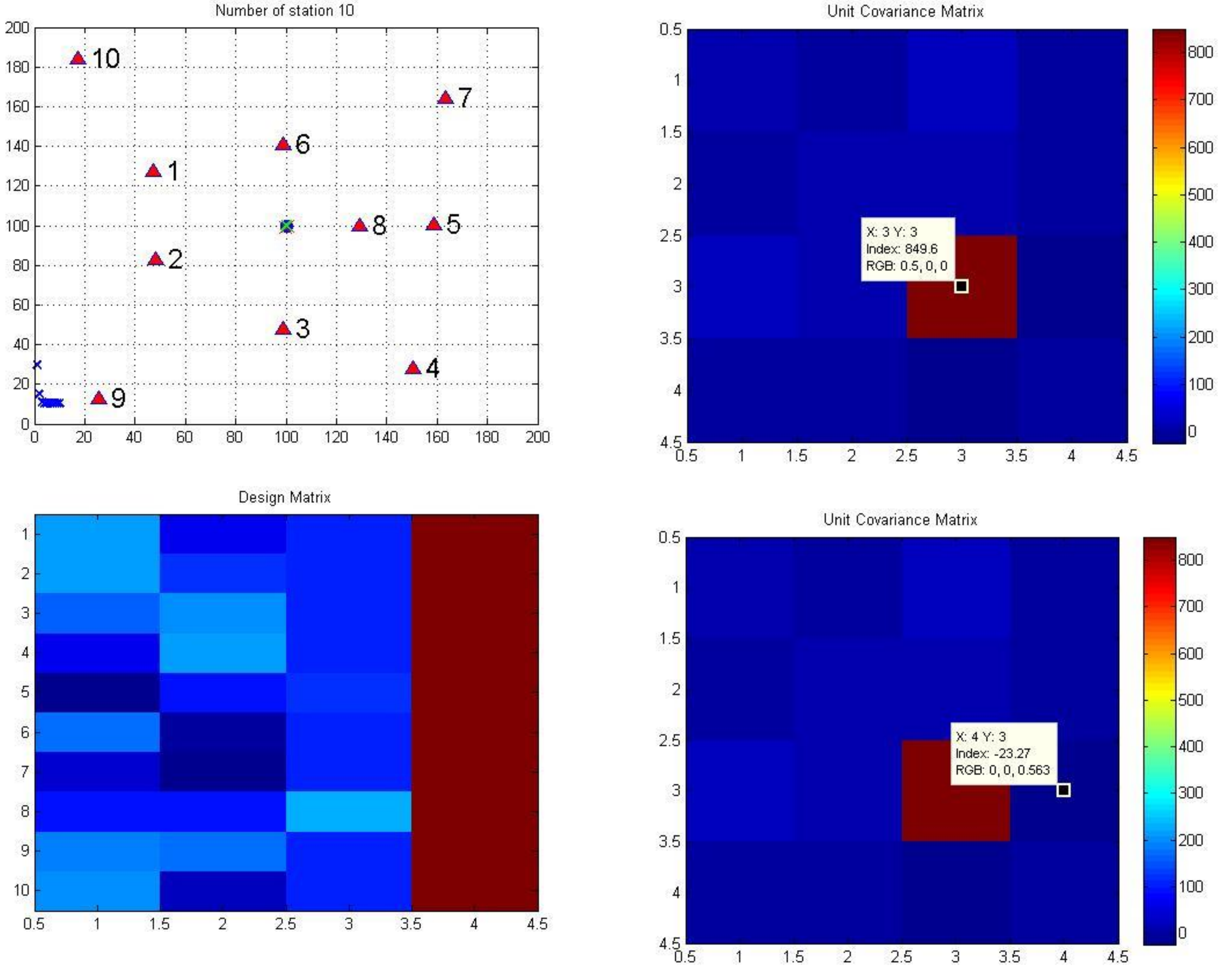
$$\sigma_X^2 = \begin{Bmatrix} \sigma_{xx}^2 & \sigma_{xy}^2 & \sigma_{xz}^2 & \sigma_{xt}^2 \\ \sigma_{yx}^2 & \sigma_{yy}^2 & \sigma_{yz}^2 & \sigma_{yt}^2 \\ \sigma_{zx}^2 & \sigma_{zy}^2 & \sigma_{zz}^2 & \sigma_{zt}^2 \\ \sigma_{tx}^2 & \sigma_{ty}^2 & \sigma_{tz}^2 & \sigma_{tt}^2 \end{Bmatrix}. \tag{20}$$

The diagonal elements are variances of the location parameters x, y, z and t_0 while the off diagonal elements give the coupling between the errors in the different hypocentral parameters. For more details, see e.g., Stein (1991). The nice property about σ_X^2 is that it is simple to calculate:

from New Manual of Seismological Observatory Practice

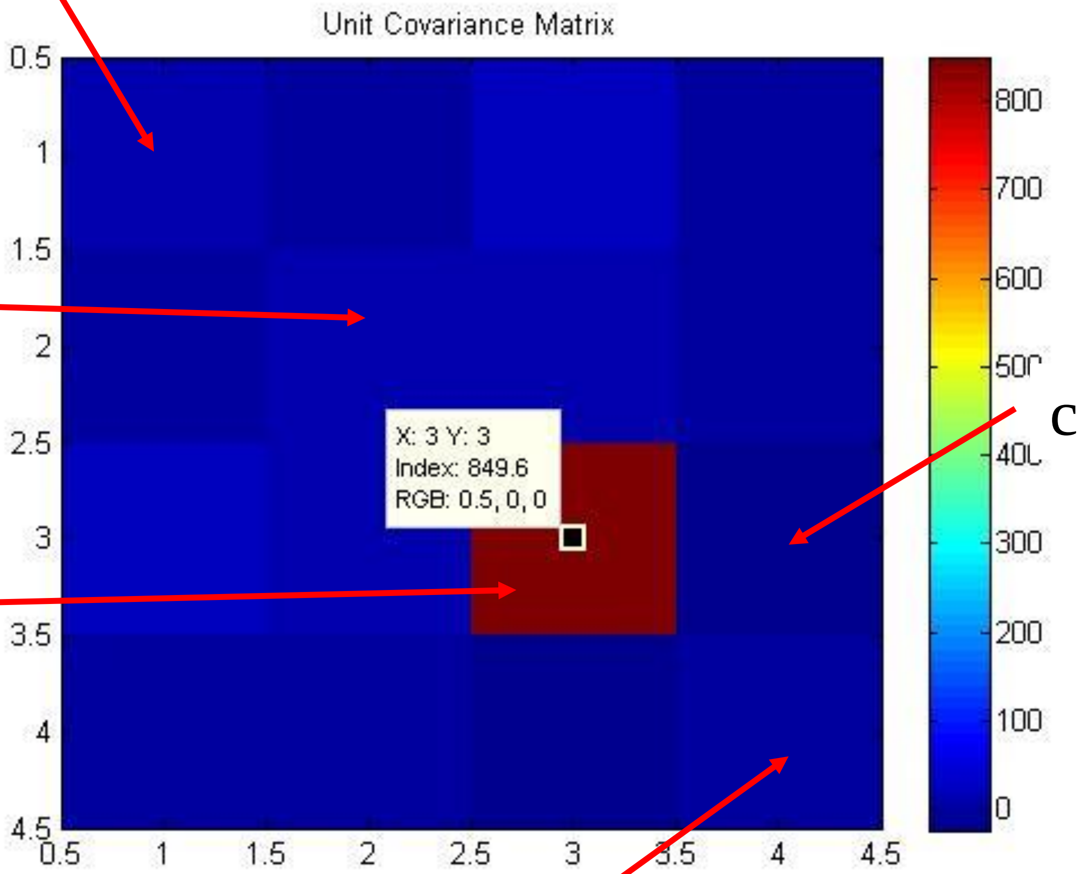
$$\sigma_X^2 = \sigma^2 * (\mathbf{G}^T\mathbf{G})^{-1}, \tag{21}$$

where σ^2 is the variance of the arrival times multiplied by the identity matrix and $\mathbf{G}^T$ is $\mathbf{G}$ transposed. The standard deviations of the hypocentral parameters are thus given by the square root of the diagonal elements and these are the usual errors reported. So how can we use the off diagonal elements? Since σ_X^2 is a symmetric matrix, a diagonal matrix in a coordinate system, which is rotated relatively to the reference system, can represent it. We now only have the errors in the hypocentral parameters, and the error ellipse simply have semi axes σ_{xx}, σ_{yy}, and σ_{zz}. The main interpretation of the off diagonal elements is thus that they define the orientation and shape of the error ellipse. A complete definition therefore requires 6 elements. Eqs. (20) and (21) also show, as stated intuitively earlier, that the shape and orientation of the error ellipse depends only on the geometry of the network and the crustal structure whereas the standard deviation of the observations is a scaling factor.


Number of station 10
1
2
3
4
5
6
7
8
9
10
0
20
40
60
80
100
120
140
160
180
200
Unit Covariance Matrix
X: 3 Y: 3
Index: 849.6
RGB: 0.5, 0, 0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
0
100
200
300
400
500
600
700
800
Design Matrix
Unit Covariance Matrix
X: 4 Y: 3
Index: -23.27
RGB: 0, 0, 0.563

the covarinace matrix depends on the source-to-station geometry (and velocity structure)

σ_{xx}=sqrt(0.2^2*7.96)=0.5km



σ_{YY}=0.6km

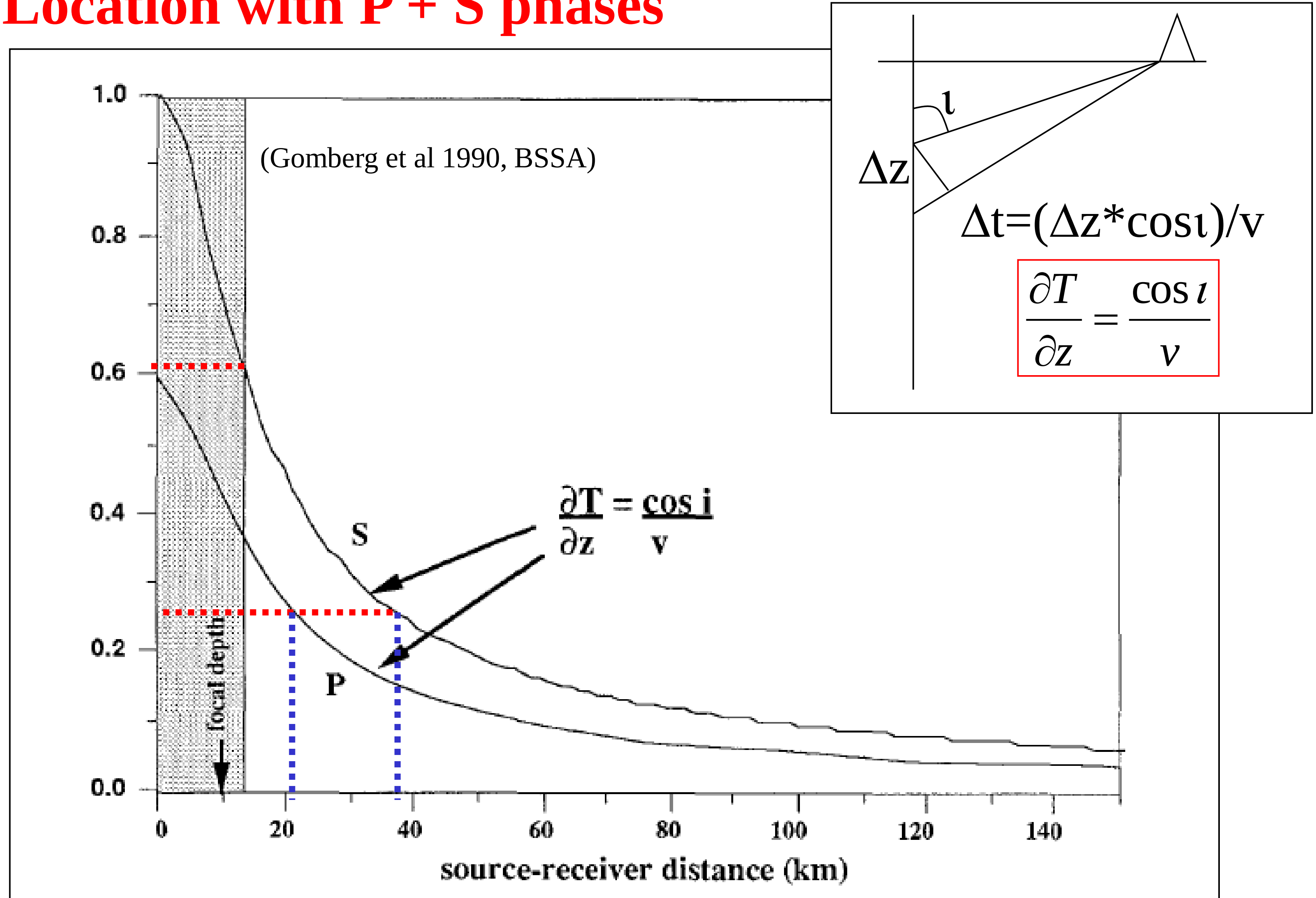
c_{ZT}=-23.27

Inverse trade-off

σ_{ZZ}=5.8km

σ_{TT}=0.17sec

Location with P + S phases



(Gomberg et al 1990, BSSA)

$$\frac{\partial T}{\partial z} = \frac{\cos i}{v}$$

$$\Delta t=(\Delta z*\cos\iota)/v$$

$$\frac{\partial T}{\partial z} = \frac{\cos \iota}{v}$$

FIG. 2. Partial derivatives of travel time with respect to focal depth for *P* and *S* phases in a homogeneous half-space. A focal depth of 10 km is used. The derivatives are normalized so that the *S* derivative has a peak value of 1 (thus, the vertical axis is dimensionless). The shaded region indicates the distance range in which *S* provides a unique constraint. $\partial T/\partial z$ = partial derivative; T = time; z = depth; i = take-off angle (up-going ray with respect to vertical); v = P or S velocity.

Location with P + S phases

Fig. 2). For up-going rays, the cosine terms of the partial derivatives are

$$\cos i = z/\sqrt{z^2 + D^2} \tag{8}$$

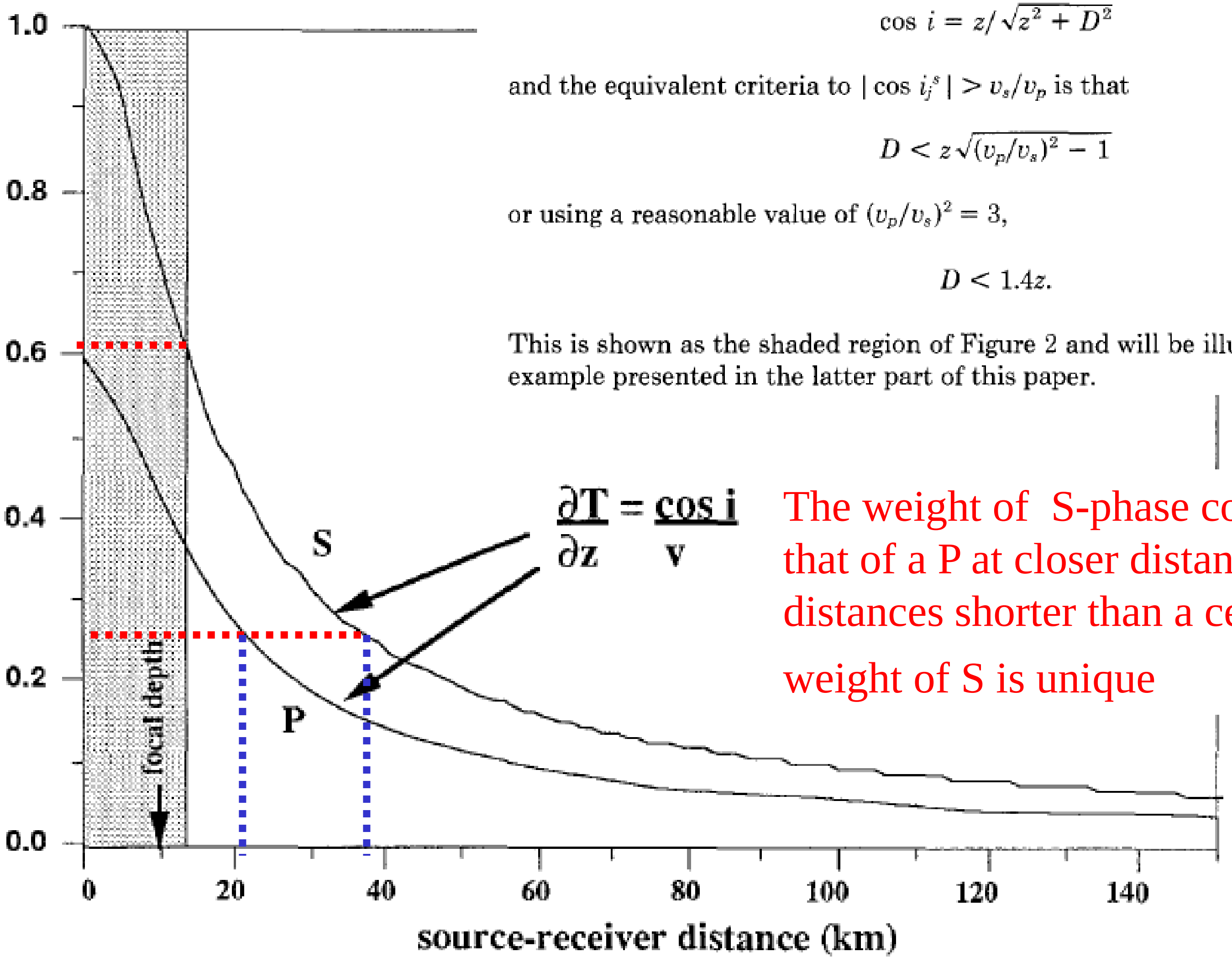
and the equivalent criteria to $|\cos i_j^s| > v_s/v_p$ is that

$$D < z\sqrt{(v_p/v_s)^2 - 1} \tag{9}$$

or using a reasonable value of $(v_p/v_s)^2 = 3$,

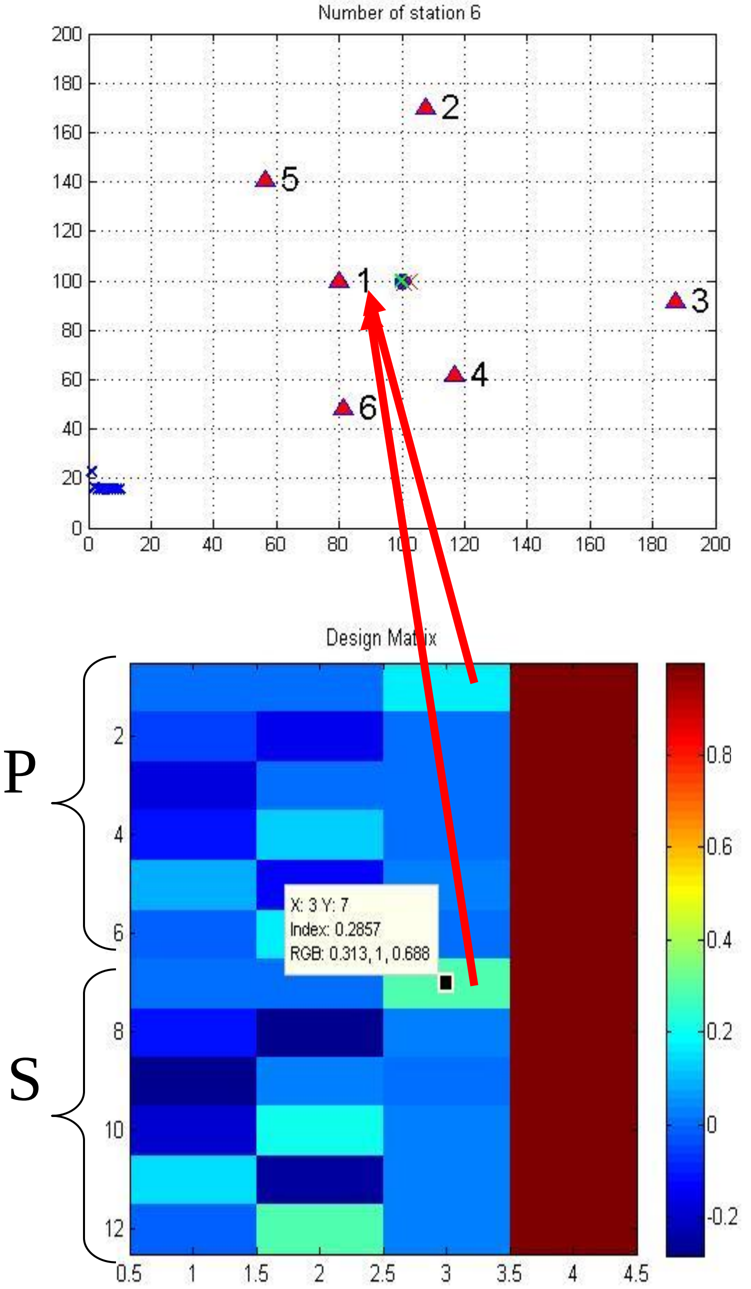
$$D < 1.4z. \tag{10}$$

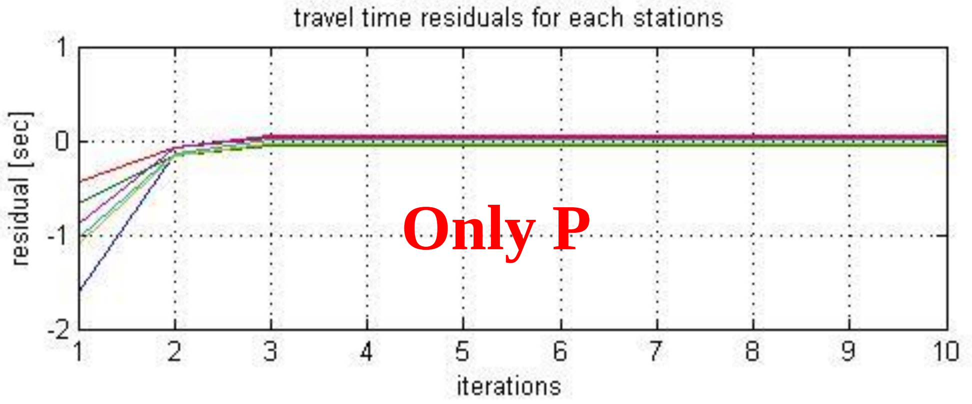
This is shown as the shaded region of Figure 2 and will be illustrated further in an example presented in the latter part of this paper.



The weight of S-phase correspond to that of a P at closer distance. At distances shorter than a certain value the weight of S is unique

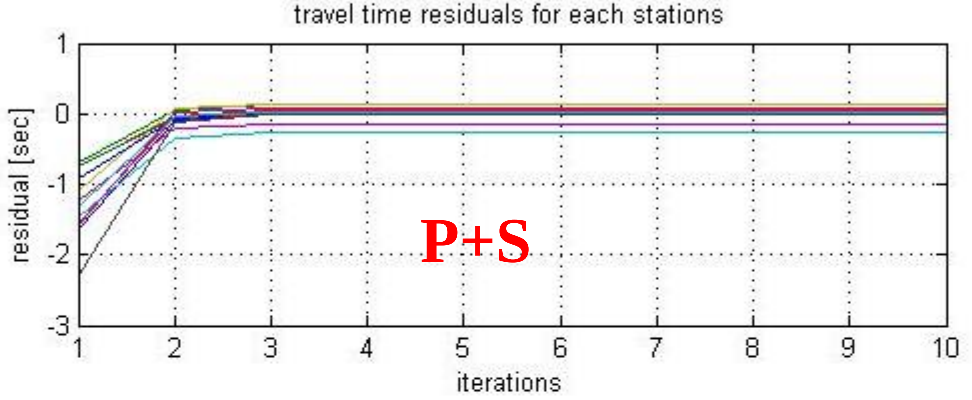
FIG. 2. Partial derivatives of travel time with respect to focal depth for P and S phases in a homogeneous half-space. A focal depth of 10 km is used. The derivatives are normalized so that the S derivative has a peak value of 1 (thus, the vertical axis is dimensionless). The shaded region indicates the distance range in which S provides a unique constraint. $\partial T/\partial z$ = partial derivative; T = time; z = depth; i = take-off angle (up-going ray with respect to vertical); v = P or S velocity.





True Model:(100.000 100.000 15.000 [km]; 0.000 [sec])

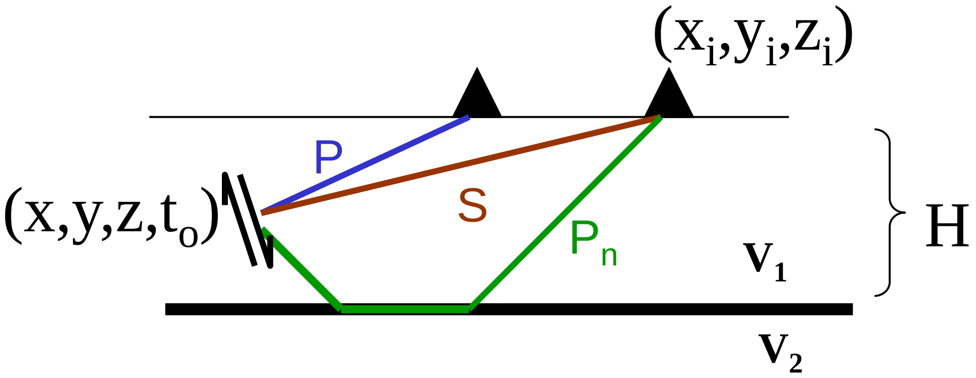
Final Model:(100.900 99.189 10.406 [km]; 0.128 [sec])



True Model:(100.000 100.000 15.000 [km]; 0.000 [sec])

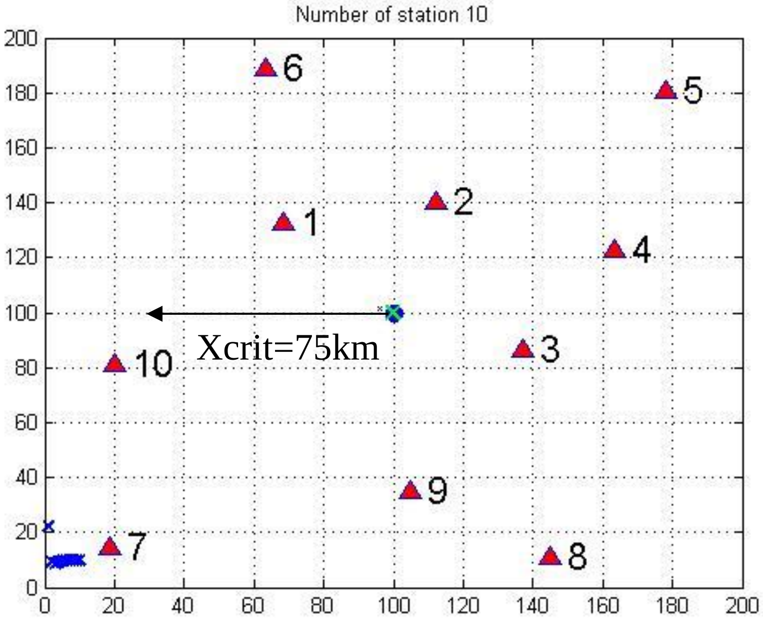
Final Model:(99.911 99.875 15.657 [km]; -0.086 [sec])

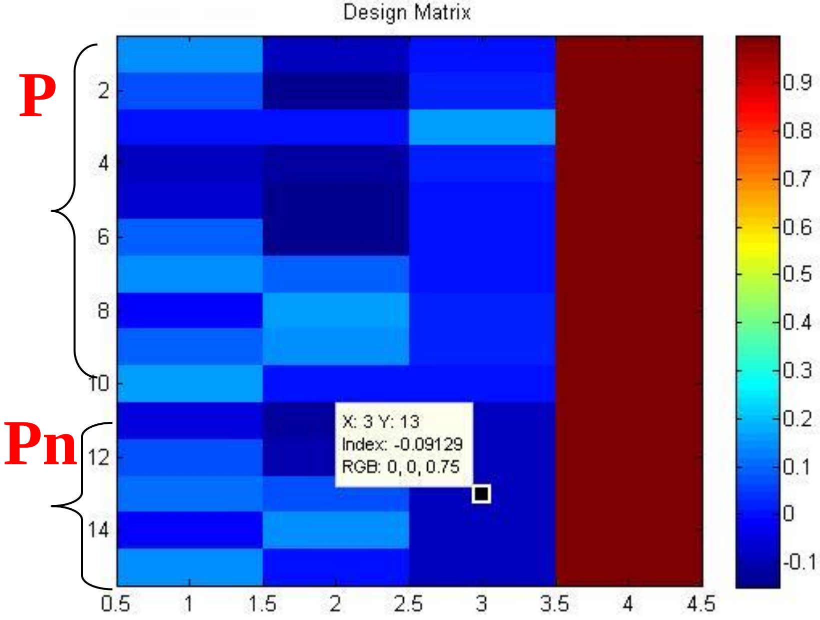
Location with P + Pn phases

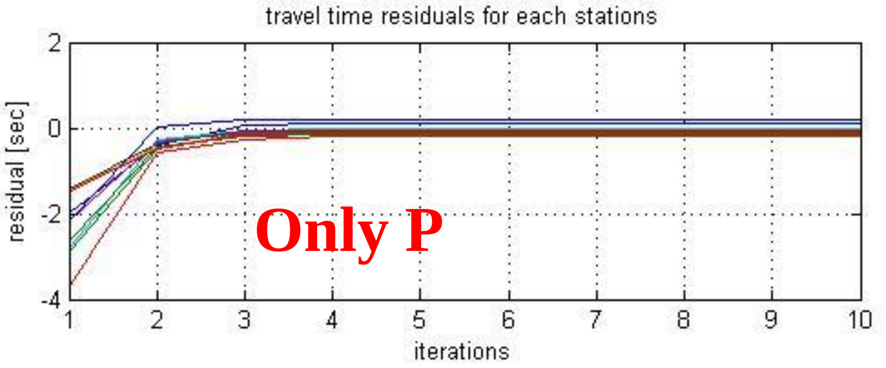


Increasing the depth, the travel time for direct P increases while the travel time for Pn decreases

$$\boxed{t_{Pn} = \frac{\left((x-x_i)^2 + (y-y_i)^2\right)^{1/2}}{v_2} + \frac{(2H-z)\left(v_2^2 - v_1^2\right)^{1/2}}{v_1 v_2} + t_0}$$

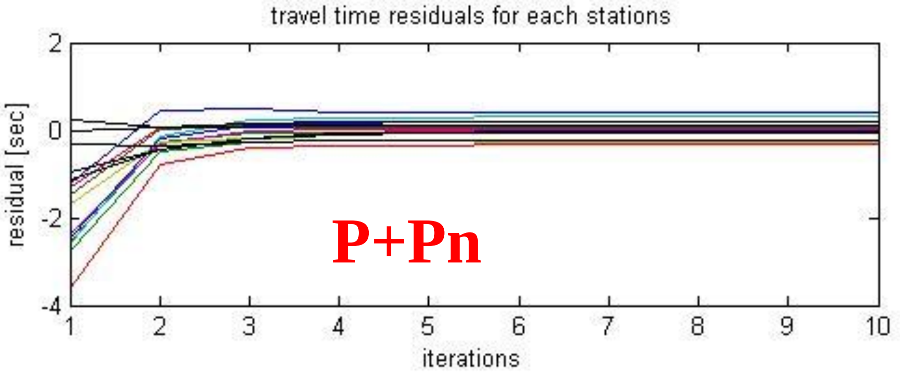






True Model:(100.000 100.000 10.000 [km]; 0.000 [sec])

Final Model:(100.506 99.776 1.000 [km]; 0.203 [sec])

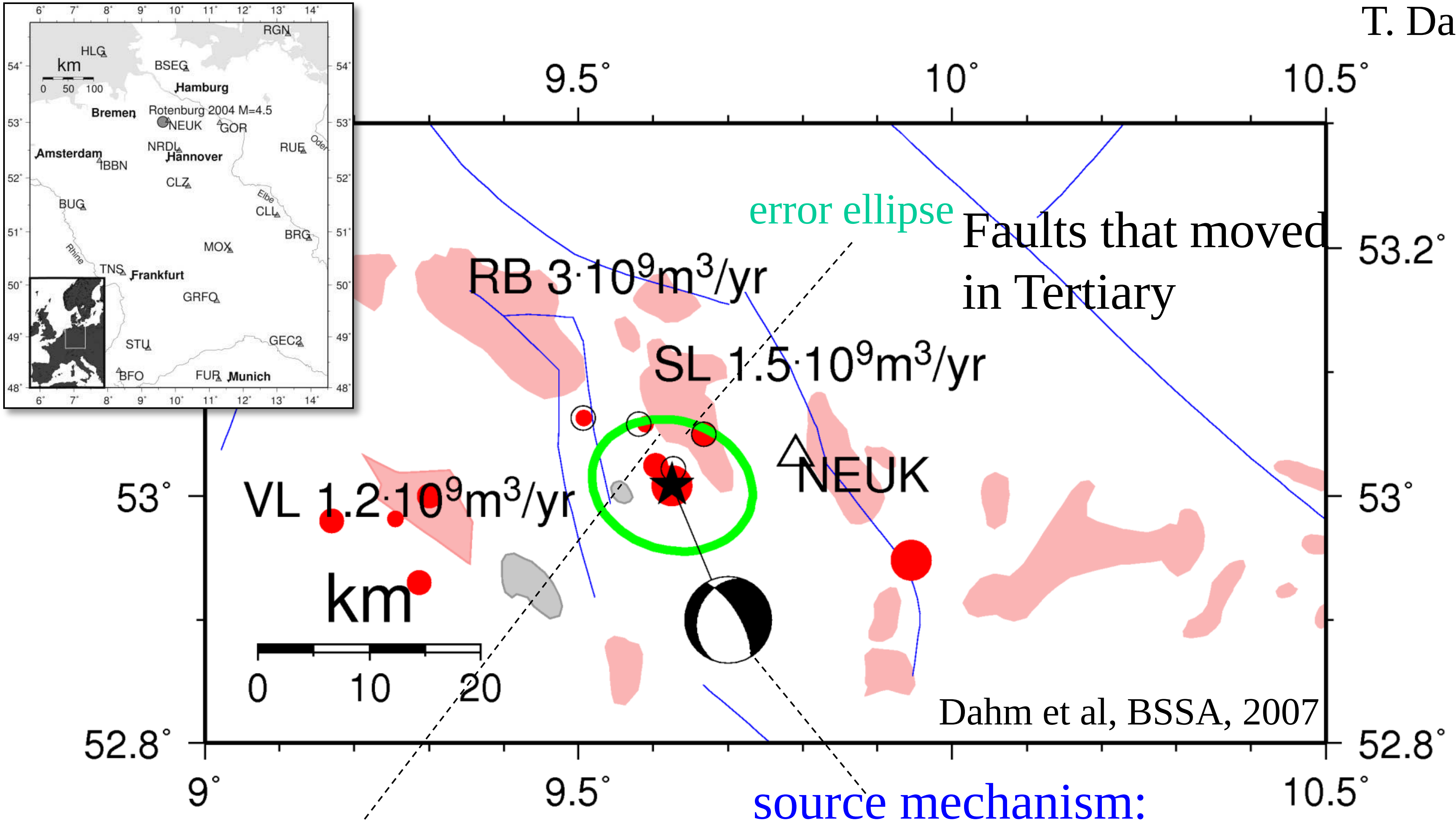


True Model:(100.000 100.000 10.000 [km]; 0.000 [sec])

Final Model:(99.973 100.094 9.658 [km]; 0.058 [sec])

The Mw 4.4 Rotenburg 2004 earthquake

T. Dahm



error ellipse

Faults that moved in Tertiary

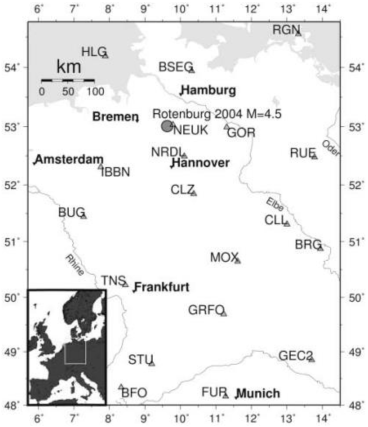
Dahm et al, BSSA, 2007

source mechanism: oblique normal faulting

production period > 20 yr

pore pressure drop > ≈ 10 MPa

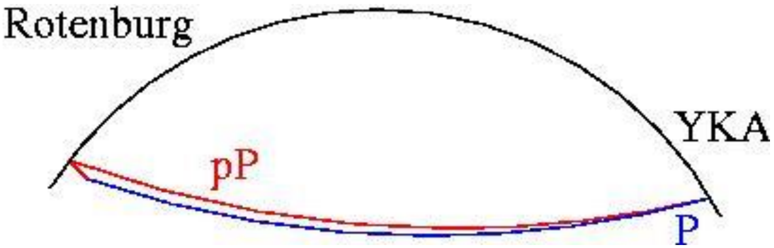
reservoir depth ≈ 4.8 km, our depth ≈ 5-6.5 km “first” depth: 12km



6° 7° 8° 9° 10° 11° 12° 13° 14°
48° 49° 50° 51° 52° 53° 54°
km
0 50 100
RGN
HLG
BSEG
Hamburg
Bremen
Rotenburg 2004 M=4.5
NEUK
GOR
NRDL
RUE
Oder
Amsterdam
IBBN
Hannover
CLZ
Elbe
CLL
BUG
BRG
Rhine
MOX
TNS
Frankfurt
GRFO
STU
GEC2
BFO
FUR
Munich

Depth phases to improve depth resolution

Earthquake depths are constrained best from rays propagating vertical



1. Epicentral stations
2. Stations far away from the epicentre, measuring depth phases, i.e. t_{pP} $-t_P$

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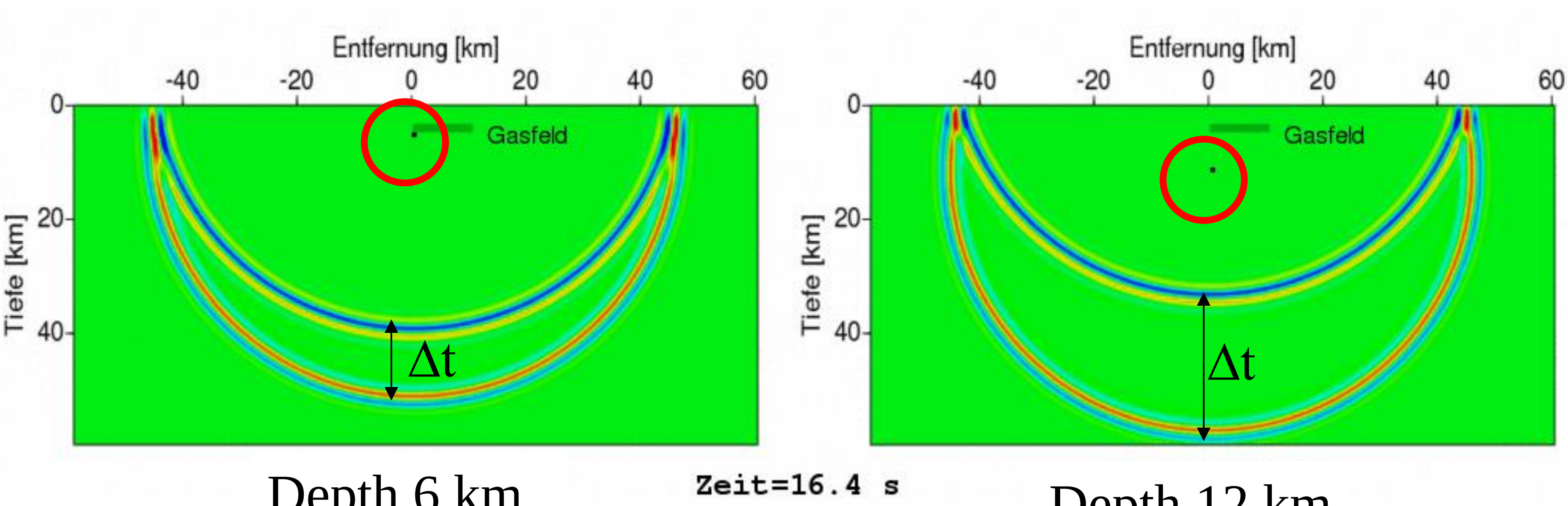
Time distance t_{pP}–t_P increases with event depth



Depth 6 km

Depth 12 km

Ray direction towards teleseismic arrays in USA and Canada

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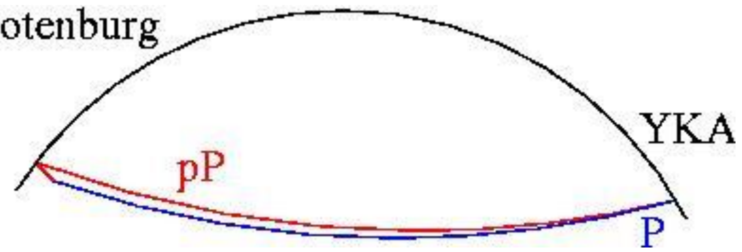
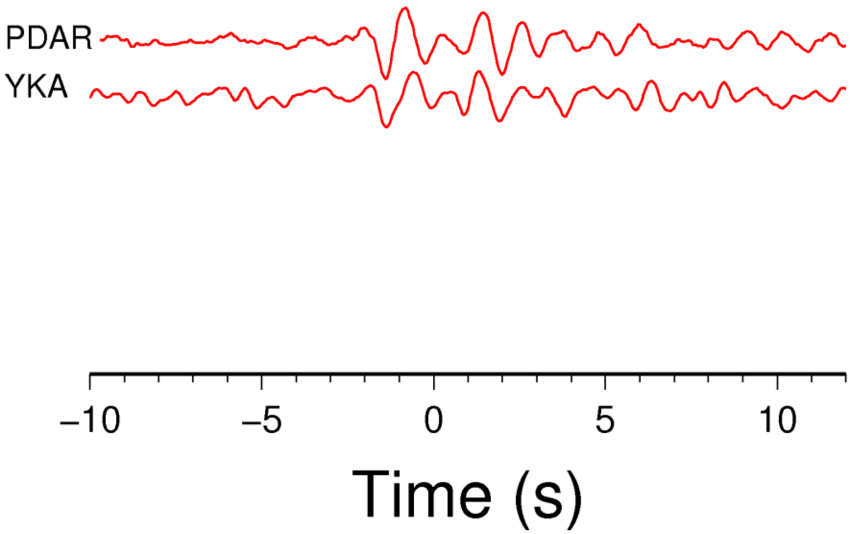
Depth-phases measured on array beams in Kanada and USA





depth phases in array beams

✓ better SNR at high freq.

✓ azimuth of arrival verified

✓ incidence angle verified

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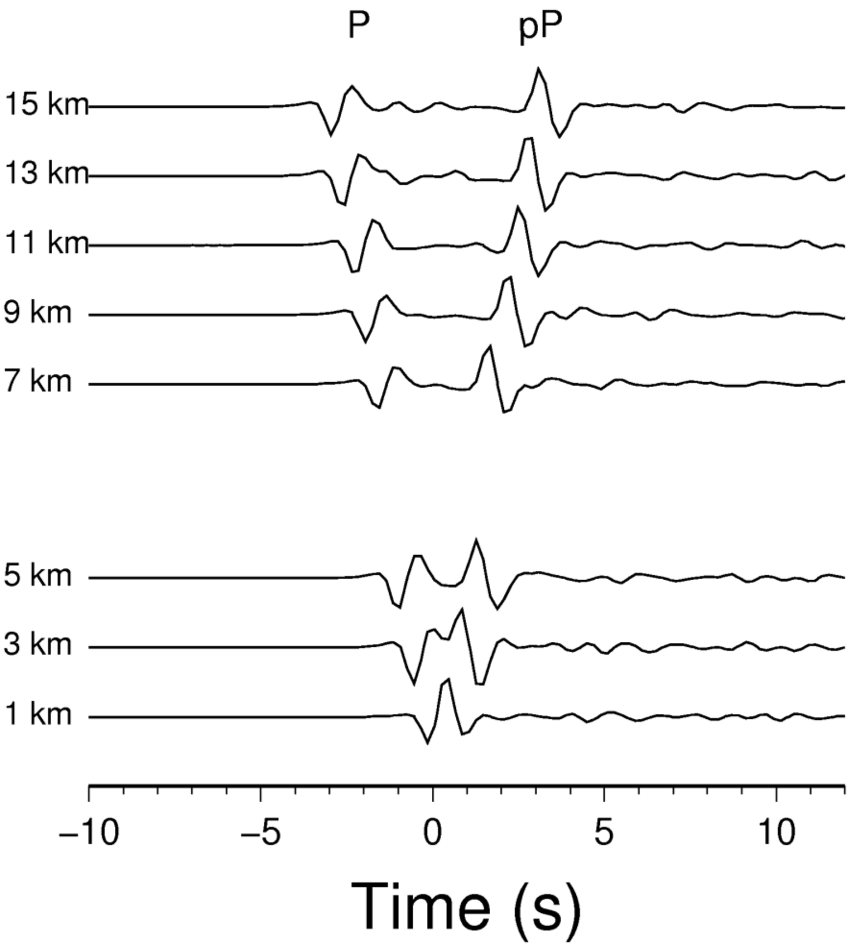
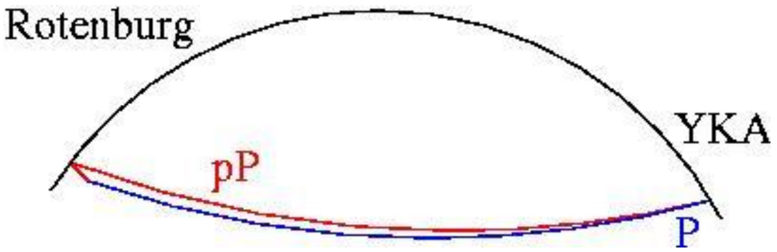
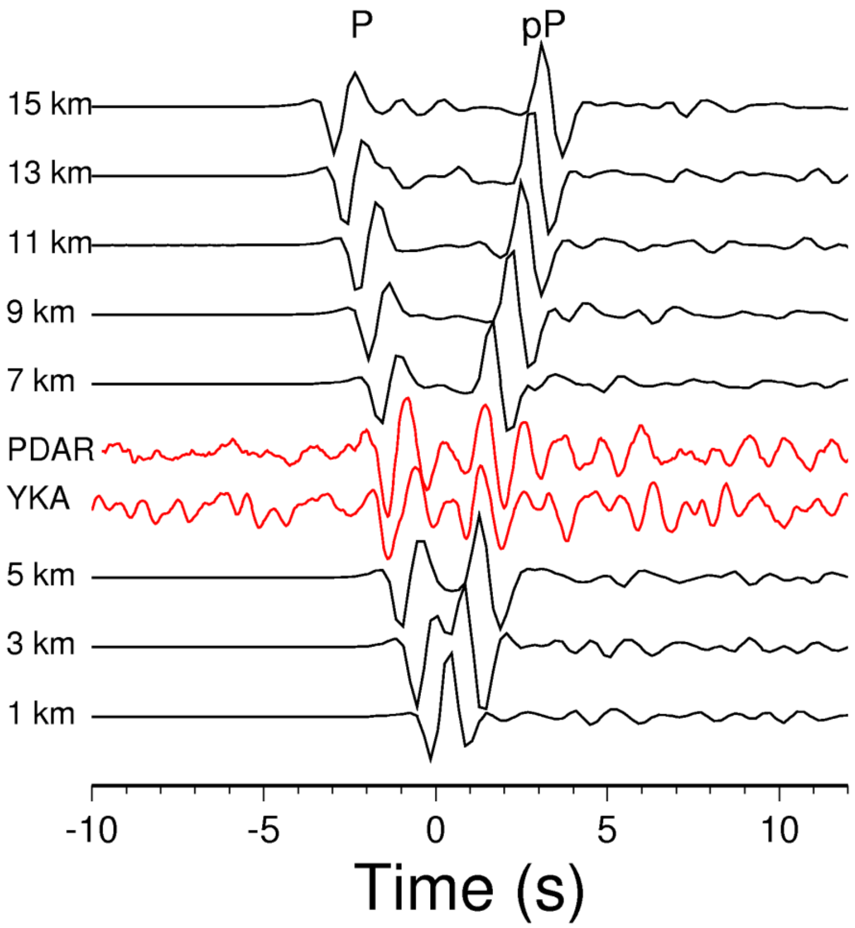
Theoretical high frequency seismograms as a function of depth





T. Dahm

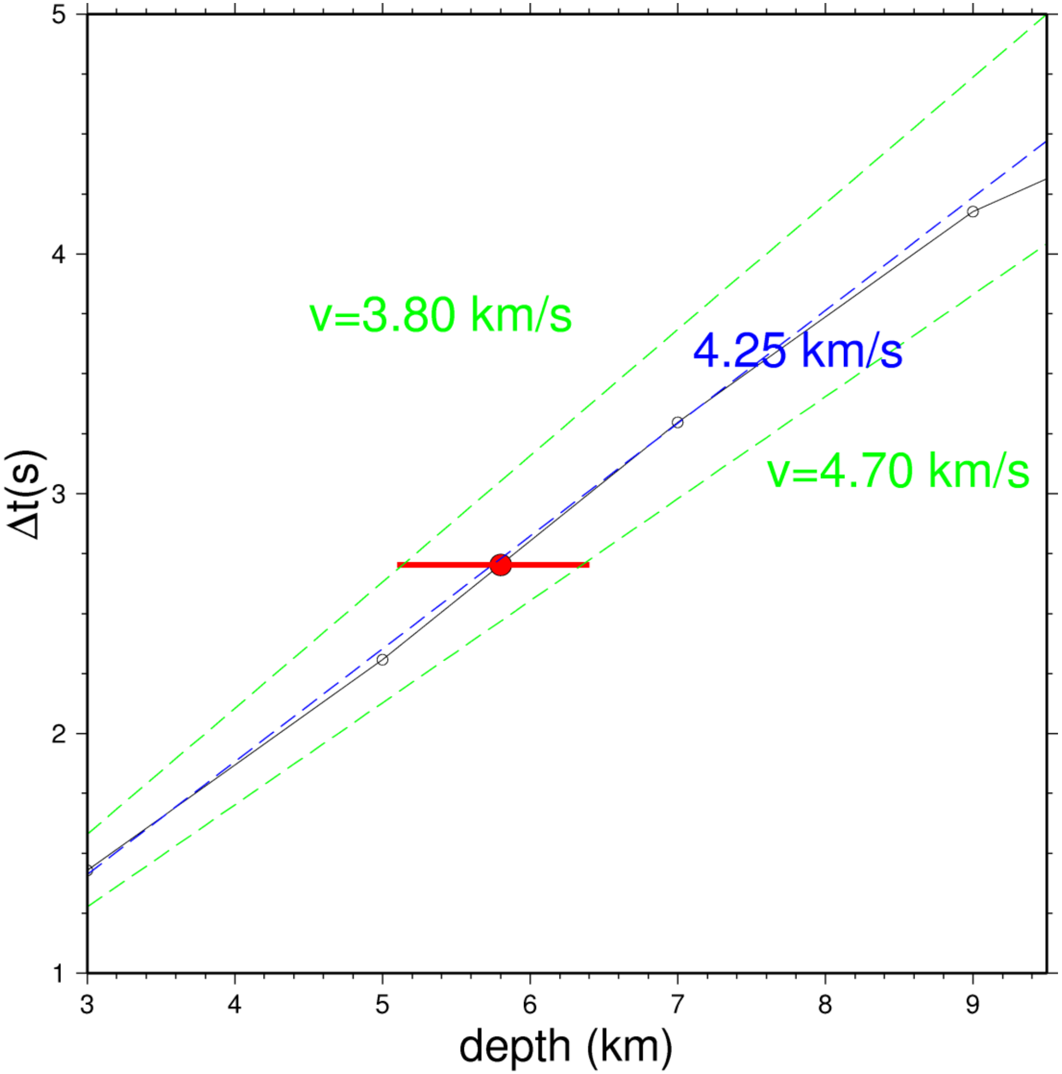
Comparison with observations



best depth at ≈5.8 km

T. Dahm

Uncertainty from unknown shallow velocities

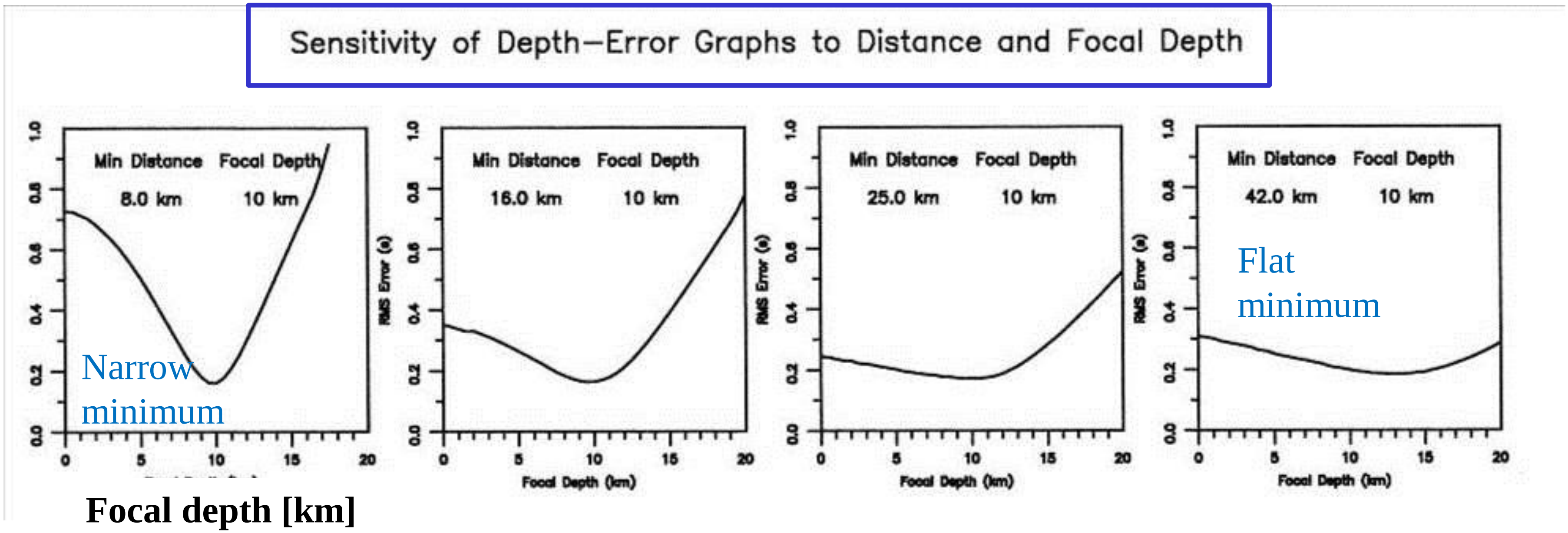


The depth uncertainty depends on the average P wave velocity in the uppermost 5 km of the crust.

Extremal velocities are v=3.8km/s and 4.7 km/s.

The depth range is between 5.1 and 6.4 km.

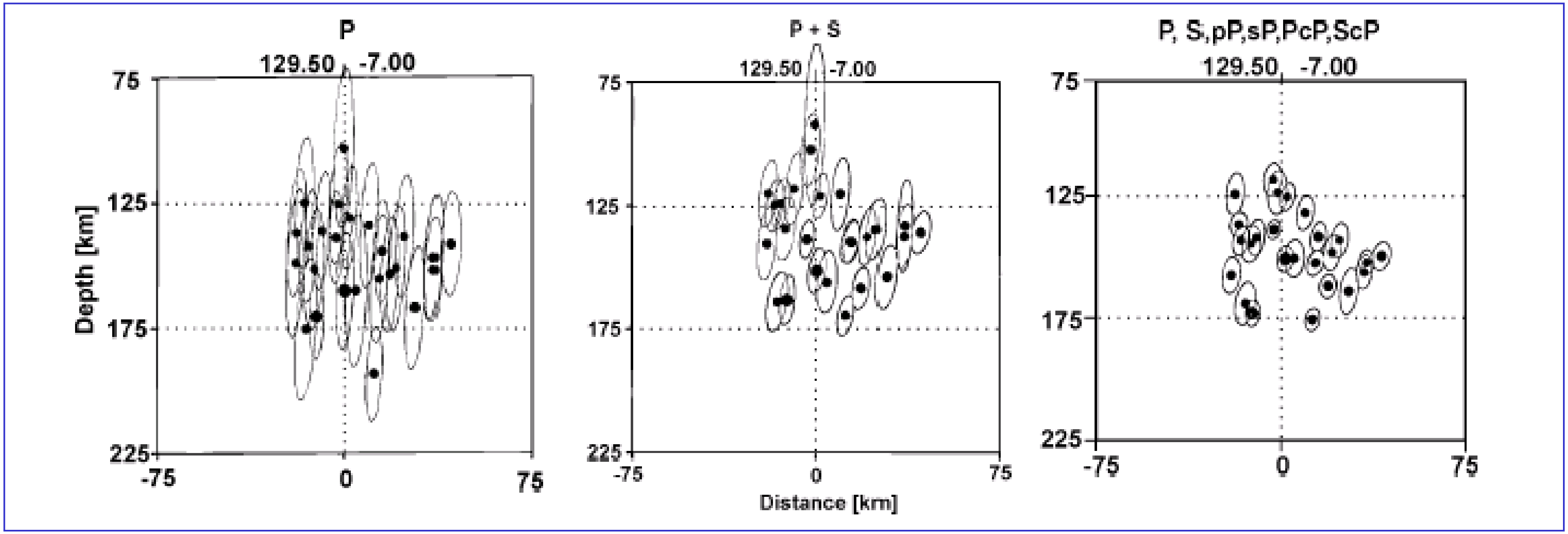
T. Dahm



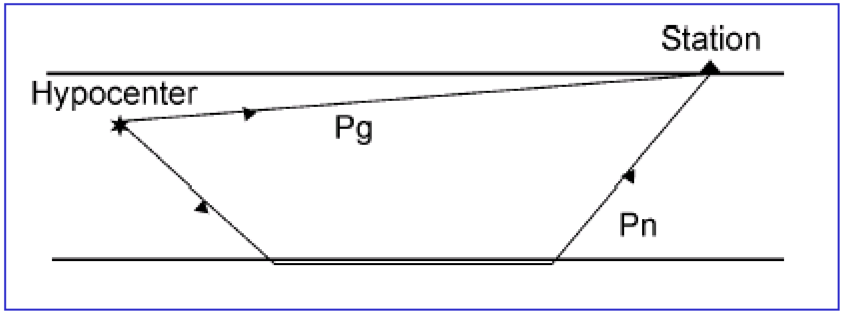
- Synthetic tests of variation in depth resolution - used in designing the network.
- As the distance for the quake to the nearest station increases the network becomes insensitive to the depth of the event (which was 10km for this test data).

courtesy of Robert Mereu and of A. Kelly

Depth can be better constrained using also S (since it has a velocity lower than P) or depth phases



Phases having travel time partial derivative with depth of opposite sign (e.g Pg and Pn) improve the depth accuracy.



Don’t forget the role played by the velocity model !!!!!!

bias in the locations!

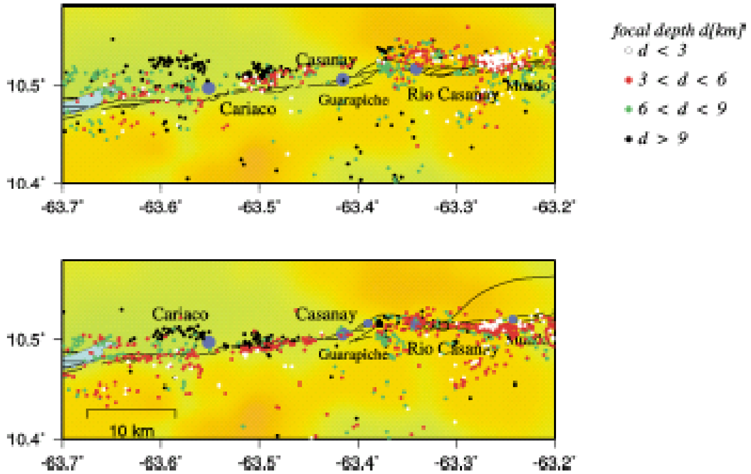
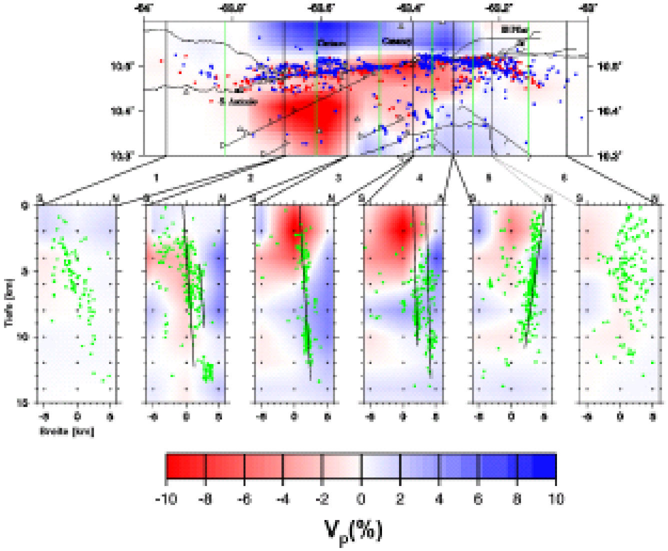
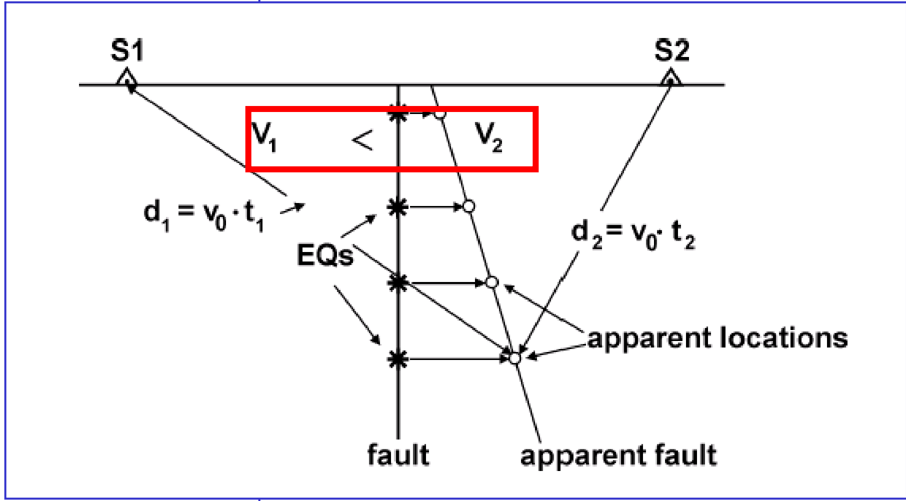


Figure 11 Epicentral distribution of aftershocks of the Cariaco earthquake (Ms=6.8) on July 9, 1997 in NE Venezuela. Top: results from HYPO71 based on a one-dimensional velocity-depth distribution. Bottom: Relocation of the aftershocks on the basis of a 3-D model derived from a tomographic study of the aftershock region (courtesy of M. Baumbach, H. Grosser and A. Rietbrock).



The velocity model can play a fundamental role in regions with strong heterogeneities



from New Manual of Seismological Observatory Practice

Table 1 gives an example of locating an earthquake with 10 stations in a model with constant velocity (from Stein, 1991). The stations are from 11 to 50 km from the hypocenter. The earthquake has an origin time of 0 s at the point (0, 0, 10) km. The starting location is at (3, 4, 20) km at 2 s. The exact travel times were calculated using a velocity of 5 km/s and the iterations were done as indicated above. At the initial guess, the sum of the squared residuals was 92.4 s2, after the first iteration it was reduced to 0.6 s2 and already at the second iteration, the .correct. solution was obtained. This is hardly surprising, since the data had no errors. We shall later see how this works in the presence of errors.

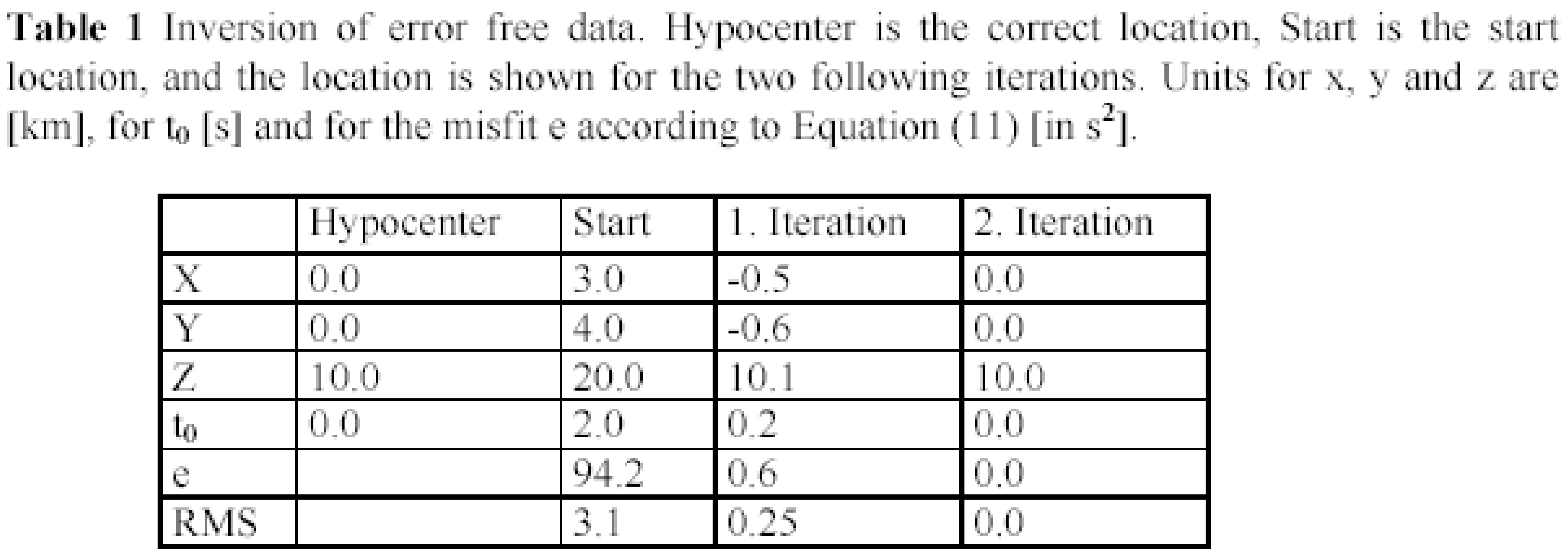


Table 1 Inversion of error free data. Hypocenter is the correct location, Start is the start location, and the location is shown for the two following iterations. Units for x, y and z are [km], for t_0 [s] and for the misfit e according to Equation (11) [in s^2].

	Hypocenter	Start	1. Iteration	2. Iteration
X	0.0	3.0	-0.5	0.0
Y	0.0	4.0	-0.6	0.0
Z	10.0	20.0	10.1	10.0
t_0	0.0	2.0	0.2	0.0
e		94.2	0.6	0.0
RMS		3.1	0.25	0.0

from New Manual of Seismological Observatory Practice

Example

We can use the previous error free example (see Table 1) and add some errors (from Stein, 1991). We add Gaussian errors with a mean of zero and a standard deviation of 0.1 s to the arrival times. Now the data are inconsistent and cannot fit exactly. As it can be seen from the results in Table 3, the inversion now requires 3 iterations (2 before) before the locations stop changing. The final location is not exactly the location used to generate the arrival times and the deviation from the correct solution is 0.2, 0.4, and 2.2 km for x, y, and z respectively, and 0.2 s for the origin time. This gives an indication of the location errors.

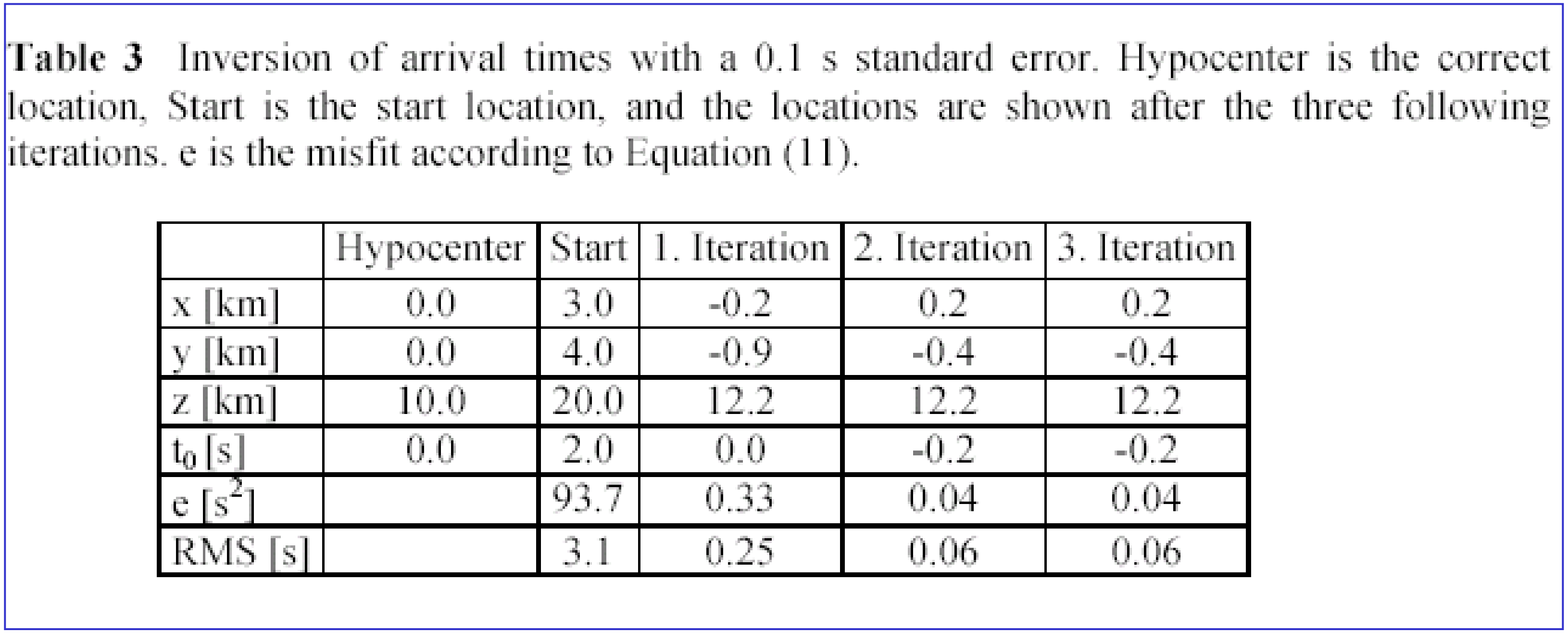


Table 3 Inversion of arrival times with a 0.1 s standard error. Hypocenter is the correct location, Start is the start location, and the locations are shown after the three following iterations. e is the misfit according to Equation (11).

	Hypocenter	Start	1. Iteration	2. Iteration	3. Iteration
x [km]	0.0	3.0	-0.2	0.2	0.2
y [km]	0.0	4.0	-0.9	-0.4	-0.4
z [km]	10.0	20.0	12.2	12.2	12.2
t_0 [s]	0.0	2.0	0.0	-0.2	-0.2
e [s^2]		93.7	0.33	0.04	0.04
RMS [s]		3.1	0.25	0.06	0.06

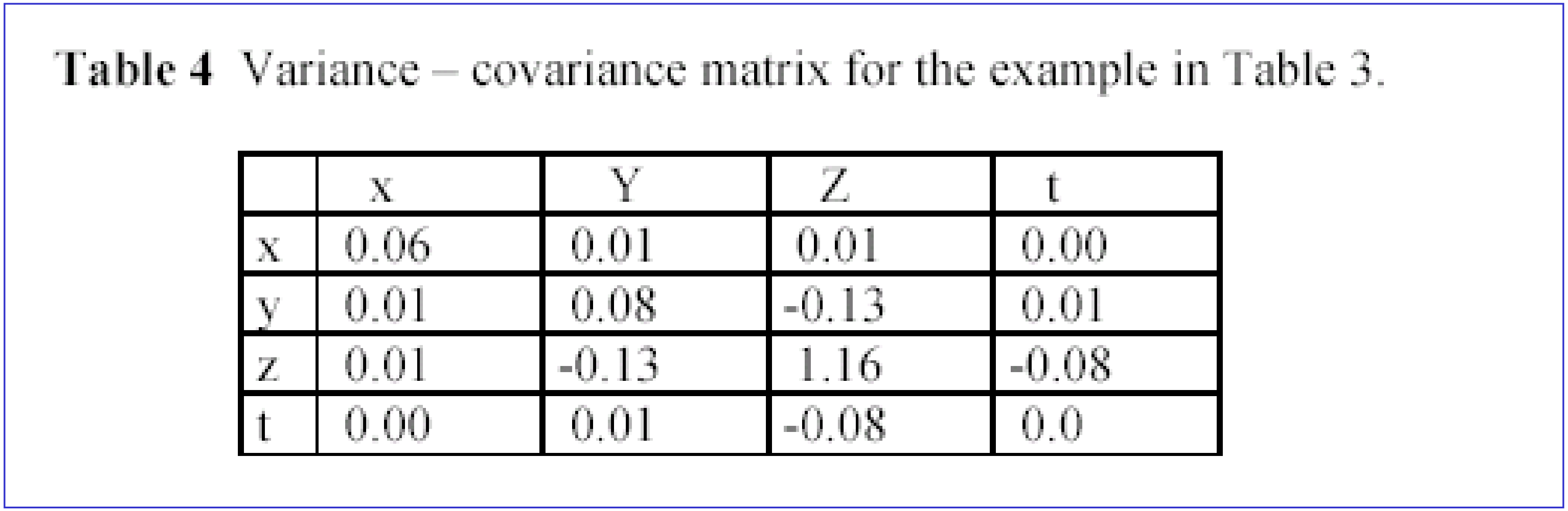
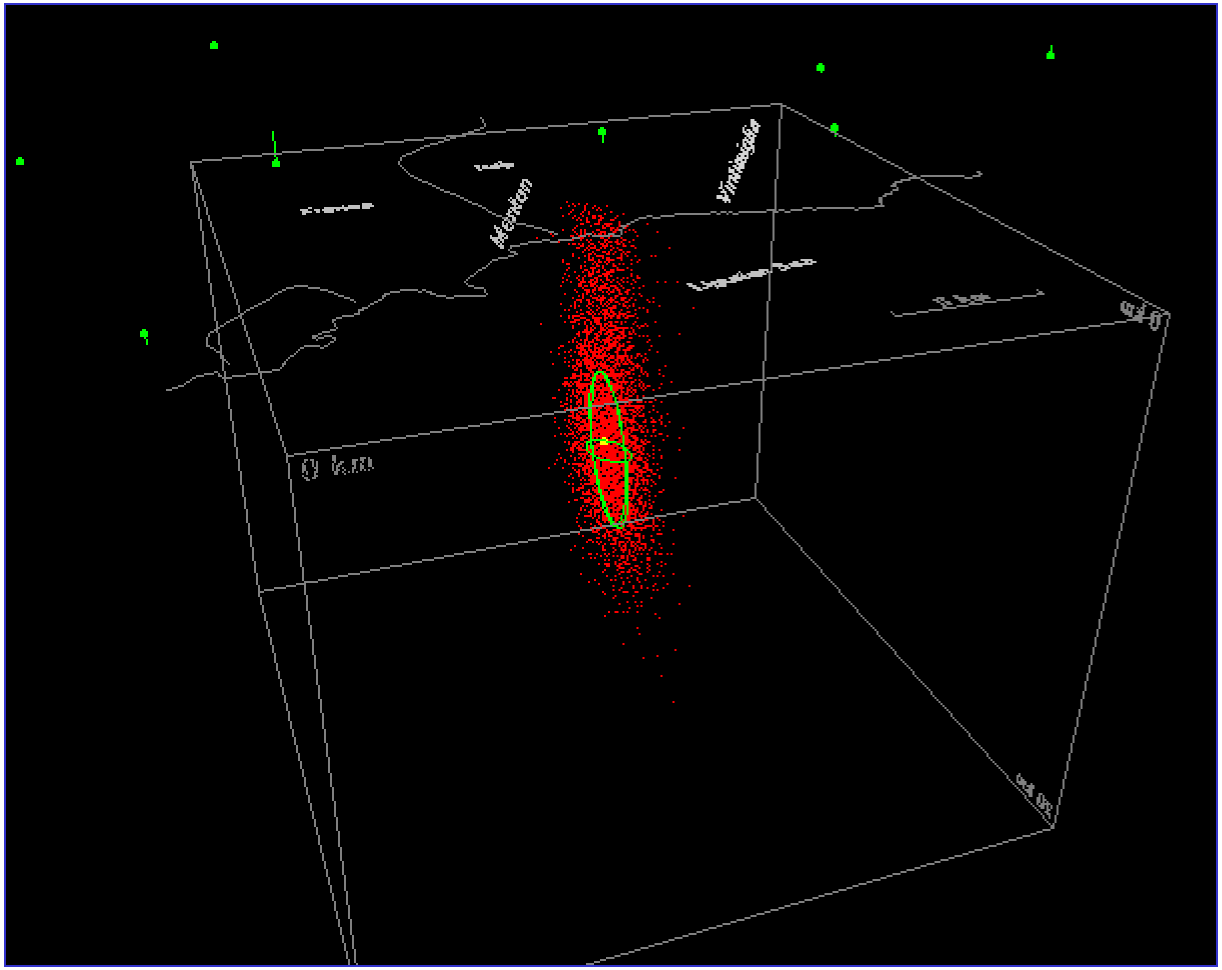


Table 4 Variance – covariance matrix for the example in Table 3.

	x	Y	Z	t
x	0.06	0.01	0.01	0.00
y	0.01	0.08	-0.13	0.01
z	0.01	-0.13	1.16	-0.08
t	0.00	0.01	-0.08	0.0

The variance . covariance matrix shows some interesting features. As seen from the dialog elements of the variance . covariance matrix, the error is much larger in the depth estimate than in x and y. This clearly reflects that the depth is less well constrained than the epicenter which is quite common unless there are stations very close to the epicenter and thus $|(d-\Delta)| / \Delta >> 1$. (rule of thumb: stations at $\Delta<2$depth) The zt term, the covariance between depth and origin time, is negative, indicating a negative trade-off between the focal depth and the origin time; an earlier source time can be compensated by a larger source depth and vice versa. This is commonly observed in practice and is more prone to happen if only first P-phase arrivals are used such that there is no strong limitation of the source depth by P times in different distances.

Antony Lomax - NONLINLOC- A probabilistic approach to earhquake location-



A useful approach when the problem is strongly non-linear and then the Geiger approach is not suitable (e.g. in regions where the velocity model is strongly heterogeneous/ anisotropic)

(see the internet page of NonLinLoc for details) adding close stations the determination of depth is improved